Name: PHY401 (Spring 2013), 02/12/13

Homework Assignment #4

(*Due Date*: Thursday, Feb. 21, 05:30 pm, in class)

4.1 *Pluto's Orbit* (cf. Ex. 4.1-3, 4.5, 4.8+9 in the textbook) *(1+3+1+3+2 pts.)*
Consider Pluto ($m_P = 3 \cdot 10^{-6} M_\odot$) in its 2-D motion around the Sun (assumed to be fixed) using Newton's universal law of gravitation, $F_G = 4\pi^2 m_P/r^2$, and Newton's 2. law of motion, in astronomical base units ($1AU$, $1y$, $M_\odot$). Use an orbit eccentricity of 0.3 ("true" value 0.248) and a semi-major axis of 40 AU ("true" value 39.5 AU).

(a) With the Sun to be located in a focal point of the elliptic orbit, use Kepler's laws and the elliptic orbit formula to calculate analytically the perihelion, aphelion, the 2 pertinent speeds and the period of Pluto's motion.

(b) Construct a FORTRAN code using the Euler-Cromer algorithm to describe Pluto's motion around the Sun (use a time-step width no larger than 0.1% of the presumed period). Choose your initial condition for Pluto on the positive x-axis with the velocity pointing in positive y direction, and the Sun fixed at the origin. Attach the code and plots of the trajectory in the x-y-plane. Verify the period, semi-major and -minor axes (measuring them from your plots "by hand") and the Sun's location in a focal point.

(c) Plot the kinetic, potential and total energy, as well as angular momentum, L, as a function of time over a few periods. Compute L from its definition via a vector product in euclidean coordinates, $\vec{L} \equiv \vec{r} \times \vec{p}$.

(d) In part (b), include Neptune's motion ($M_{Ne} = 5 \cdot 10^{-5} M_\odot$) via a coplanar circular orbit of radius 30 AU starting on the negative x-axis neglecting the mutual force between Neptune and Pluto in the computation of the orbits. Over a time span of 100000 years, what is the distance of closest approach between the 2 planets and the corresponding (would-be) percentage distortion that Neptune exerts on Pluto (relative to the Sun's force)? Increase Neptune's mass to $M_{Ne} = 0.1 M_\odot$ and recalculate Pluto's orbit including the force due to Neptune (keep the Sun fixed; can you neglect Pluto's force on Neptune?); comment on your results.

(e) Back to the 2-body problem, i.e., no Neptune: Modify the inverse-square force law by changing the exponent from -2 to -2.1 and plot Pluto's orbit over many periods using the same initial conditions as in (b); comment on your results (e.g. precession period). Do another run with a slightly different initial condition (e.g. increasing v_0 by 0.1%) and plot the distance $s(t) = |\vec{r}_1(t) - \vec{r}_2(t)|$ between the 2 trajectories; interpret your finding.