Name: PHY401 (Spring 2013), 01/24/13

Homework Assignment #2

(*Due Date*: Thursday, January 31, 05:30 pm, in class)

2.1 *Medieval Castle Defense* *(2+3+3+2 pts.)*

A castle is built on the hills of a river valley, $h = 60m$ above the river/valley level. The castle's horizontal distance (from the bottom of the hill) to the nearside river bank is $180m$, and the river is another $55m$ wide. The castle is equipped with several cannons which can eject solid smooth rock spheres of $25\,cm$ diameter at a speed of $v_0 = 45m/s$ (the rock's mass density is $2800\,kg/m^3$). The castle knights are about to fire the cannons in view of 500 enemy troops approaching the far-side river bank, but they have to figure out the appropriate launch angle, Θ_0. To save precious gunpowder, the court jester suggests to perform some estimates prior to the first shot.

(a) Analytical (benchmark) estimates: Neglecting air drag, derive the analytical expression for the horizontal range of the projectile as a function of Θ_0, v_0, h and g $(= 9.8m/s^2)$. Use your pocket calculator to obtain the projectile range, $R(\Theta_0) = x_{\max}(\Theta_0)$, for a few launch angles between 30° and 45° to roughly estimate the maximal theoretical range, $R_{\max} = R(\Theta_0^{\max})$, in vacuum. Sketch the trajectories in a hand-drawn graph (no need for accuracy except for Θ_0 and R).

(b) Write a FORTRAN code to compute the trajectory including a quadratic (in speed) air drag with a drag coefficient of 0.5 for the cannon ball (air density $\rho_{\text{air}} = 1.29kg/m^3$). Can the cannon ball reach the far-side river shore? Attach the source code and a plot of trajectories including the one with the maximal range, $R_{\max}$. Compute and plot the cannon ball's speed upon impact, as function of Θ_0.

(c) How accurate does the launch angle have to be to hit a target of $2m$ horizontal size (neglect vertical size) which has just moved fully into the maximal range (i.e., its center is at $R_{\max} - 1m$)? How large is the angular variation to hit the target when it has moved to 50% of the maximal range (evaluate both solutions, i.e., below and above $\Theta_0^{\max}$)? Use a linear interpolation technique in determining all impact locations.

(d) Suddenly a horizontal head wind of $20mph$ starts to blow, bringing also heavy rain fall; repeat part (b) for these conditions (approximate the effect of the rain drops by a 10% increase of the air density).