Name: PHY401 (Spring 2010), 04/23/10

Homework Assignment #9

(*Due Date*: Monday, May 03, 10:20 am, in class)

9.1 *2-D Percolation and Critical Exponent* (cf. Ex. 7.27+7.31 in the textbook) *(3+2 pts.)*
Write a FORTRAN program to simulate the percolation transition on a $N \times N$ square lattice by subsequently filling lattice sites randomly, corresponding to an increase in occupation probability, p.

(a) Determine the critical probability p_c by checking, after each new entry, for the appearance of a spanning cluster, and plot the latter when it first appears (once for each value of N). Perform this procedure for various lattice sizes N (e.g. N=5,10,15,20,30,50,80) using an average over ca. 50 simulations for each N, and plot $p_c(N^{-1})$ to extrapolate to the infinite-size limit, $p_c(0)$.

(b) For fixed lattice size (as large as possible, e.g., N=100), compute the fraction

$$F(p > p_c) \equiv \frac{\text{no. of sites in spanning cluster}}{\text{no. of occupied sites}} \tag{1}$$

as a function of p above the critical p_c. Average your results for each p over ca. 50 simulations. Fit your results to a power-law ansatz

$$F = F_0 (p - p_c)^{\beta} \tag{2}$$

by plotting the logarithm of both sides and extracting the slope of a straight-line fit (note that the power-law only applies for p not "too far" above p_c).

9.2 *2-D Ising Model* (cf. Ex. 8.1,8.2+8.3 in the textbook) *(2+3 pts.)*
Consider the 2-D Ising Model on a square lattice with nearest-neighbor (4) interaction strength J=1.5 and at zero external magnetic field.

(a) Use the Newton-Raphson root finder algorithm to numerically solve the mean-field selfconsistency relation for the average magnetization,

$$\langle s \rangle = \tanh\left(\frac{zJ}{k_B T}\langle s \rangle\right) \ , \tag{3}$$

in the temperature range from zero to just above T_c. Compare your numerical results at large and small(!) temperature to analytic results obtained from a suitable Taylor expansion.

(b) Write a FORTRAN program to numerically study the 2-D Ising Model on a $n \times n$ lattice with periodic boundary conditions ($N = n^2$: total number of spins, $M = \sum_i s_i = N\langle s \rangle$). Calculate the (time-average) magnetization as a function of temperature (in equilibrium, i.e. after sufficient Monte Carlo sweeps to reach equilibrium) and determine the critical temperature as well as the critical exponent β in $\langle M \rangle \propto (T_c - T)^{\beta}$.
(*Hints*: to extract β, concentrate on the temperature region $T = (0.9 - 1)T_c$. For several "trial" values of T_c, plot $\log(M)$ vs. $\log(T_c - T)$ and take as a best estimate the case where a straight line appears most appropriate; perform this procedure for two lattice sizes, n=15 and 25, to check for finite-size effects.)