Name: PHY401 (Spring 2010), 03/12/10

Homework Assignment #6

(*Due Date*: Friday, March 26, 10:20 am, in class)

6.1 *Ideal Wave Propagation, Stability and Spectral Decomposition* (cf. Ex. 6.1, 6.2, 6.4, 6.9, 6.12, 6.13) *(3+2+2+3 pts.)*
Write a FORTRAN program to numerically calculate the signal propagation on a uniform string (length $L = 1m$, tension force $F_T = 1200N$, total mass $30g$, both ends fixed) using the ideal wave equation,

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \ , \tag{1}$$

for the displacement $y(x,t)$ at string position x and time t. In the discretized form of the equation, set the numerical coefficient $r \equiv c/(\Delta x/\Delta t)$ equal to one unless otherwise noted.

(a) Impart an initial Gaussian pluck on the string, $y(x, t_{0,1}) = y_0 \exp[-(x-\bar{x})^2/2\sigma_x^2]$ with $y_0 = 0.25m$, $\bar{x} = 0.3m$ from the left end of the string and a full-width-at-half-maximum of $0.1m$ (what is the corresponding value of σ_x?). Study the signal propagation by plotting time snapshots of your solution, $y(x, t_s)$, as the signal moves across the string. How are the signals reflected at the ends? Also produce a 3-D "surface" plot for y as a function of x and t (you can use, e.g., the `splot` command in gnuplot); the time axis should cover at least 1.2 times the period of the motion.

(b) Study and comment on the consequences of choosing values of $r = 0.8, 1.2$ in your numerical solution in (a).

(c) Perform a Fourier analysis of the string excitation in part (a), i.e., for a fixed position on the string (e.g. $x_s = 0.5m$), record $y(t)$ and evaluate the Fourier transform using a suitable time discretization. Plot the power spectrum vs. frequency and interpret the peaks in the spectrum in terms of the harmonics of the string. Repeat the Fourier analysis for a different position on the string (e.g. $x_s = 0.7m$) and comment on the difference to $x_s = 0.5m$.

(d) Initialize the string with a more realistic "triangular" pluck, with a maximum $y(0.3, t_{0,1}) = 0.25m$, which linearly connects to the ends (see Fig. 1 below). Study its time dependence and power spectrum in analogy to parts (a) and (c) (for $x_s = 0.5m$ only).

Fig.1:
"Realistic" Pluck