Name: PHY401 (Spring 2010), 03/01/10

Homework Assignment #5

(*Due Date*: Wednesday, Mar. 10, 10:20 am, in class)

5.1 *Poisson Equation for Dipole* (cf. Ex. 5.7) *(2+1+2 pts.)*
Write a FORTRAN program to numerically calculate the electric potential of a static electric dipole, i.e., two point charges $Q/\epsilon_0 = \pm 1$ separated by a distance $a = 0.6$. Solve the Poisson equation in Cartesian coordinates but impose a spherical boundary condition, $V(R) = 0$, at a large distance (e.g., $R = 10$).

(a) Starting from an initial condition of zero potential use the Jacobi relaxation algorithm with appropriate numerical tolerance and grid density to obtain the (converged) solution. Plot the equipotential lines. Plot your result for $V(r)$ (r: distance from origin) and compare it to the expected large-distance behavior of the dipole potential.

(b) Investigate how the number of required iteration steps, N_{iter}, increases with reducing the tolerance (error) limit, "*tol*", and plot it, i.e., $N_{iter}(tol)$.

(c) Write a second program by modifying the algorithm to using the Simultaneous Over-Relaxation (SOR) method, and allowing for different grid densities in different relaxation runs. For fixed accuracy in the solution (not total tolerance!), investigate (and plot) how the number of iteration steps, N_{iter}, depends on the number n=n_x=n_y of grid points. You should find $N_{iter} \propto n^2$ and $N_{iter} \propto n$, for the Jacobi and SOR method, respectively.

5.2 *Biot-Savart Law* *(2+2+1 pts.)*
Use Biot-Savart's Law for the magnetic field induced by a thin-wire current,

$$d\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \frac{d\vec{r}\,' \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} , \tag{1}$$

to construct a FORTRAN code calculating the $\vec{B}$-field for a quadratic current loop (side length $1m$, $\mu_0 I$=8 in SI units, running counter-clockwise) lying centered in the x-y plane. Calculate and plot the x-, y- and z-components of the magnetic field for
(i) x=y=0 as function of z; compare to (i.e., plot in the same graph) the analytic result for a circular current loop with the same area as the square;
(ii) z=1m, y=0 as a function of x;
(iii) y=0, x=0.5m as a function of z.