## Homework Assignment \#4

(Due Date: Friday, Feb. 26, 10:20 am, in class)
4.1 Pluto's Orbit (cf. Ex. 4.1-3, 4.5, 4.8+9 in the textbook) ( $1+3+1+1+2+2$ pts.) Consider a Pluto-like planet ( $m_{P}=3 \cdot 10^{-6} M_{\odot}$ ) in its 2-D motion around the Sun (assumed to be fixed) using Newton's universal law of gravitation, $F_{G}=4 \pi^{2} m_{P} / r^{2}$, and Newton's 2. law of motion, in astronomical base units ( $\left.1 A U, 1 y, M_{\odot}\right)$. Use an orbit eccentricity of 0.3 ("true" value 0.248 ) and a semimajor axis of 40 AU ("true" value $39.5 A U$ ).
(a) With the Sun to be located in a focal point of the elliptic orbit, use Kepler's laws and the elliptic orbit formula to calculate analytically the perihelion, aphelion, the 2 pertinent speeds and the period of Pluto's motion.
(b) Construct a FORTRAN code using the Euler-Cromer algorithm to describe Pluto's motion assuming the Sun to be fixed (use a time-step width no larger than $0.1 \%$ of the presumed period). Choose your initial condition for Pluto on the positive $x$-axis with the velocity pointing in positive $y$ direction, and the Sun at the origin. Attach the code and plots of the trajectory in the $x$ - $y$-plane. Verify the period, semimajor and -minor axes and the Sun's location in a focal point.
(c) Plot the kinetic, potential and total energy, as well as angular momentum, $L$, as a function of time over a few periods. Compute $L$ from its definition via a vector product in euclidean coordinates, $\vec{L} \equiv \vec{r} \times \vec{p}$.
(d) Replace the Euler-Cromer method with the Euler method, with initial conditions as in part (a). Plot and comment on the results. Does decreasing the time-step width help?
(e) In part (b), include Neptune's motion $\left(M_{N e}=5 \cdot 10^{-5} M_{\odot}\right)$ via a circular orbit of radius $30 A U$ starting on the negative $x$-axis (you can neglect the mutual force between Neptune and Pluto in the computation of the orbits). Over a time span of 100000 years, what is the distance of closest approach between the 2 planets and the corresponding (would-be) percentage distortion that Neptune exerts on Pluto (relative to the Sun's force)?
(f) Modify the inverse-square force law by changing the exponent from -2 to -2.05 and plot Pluto's orbit over many periods using the same initial conditions as in (b); comment on your results (e.g. precession period). Do another run with a slightly different initial condition (e.g. increasing $v_{0}$ by $0.1 \%$ ) and plot the distance $s(t)=\left|\vec{r}_{1}(t)-\vec{r}_{2}(t)\right|$ between the 2 trajectories; interpret your finding.

