4.1 Pluto’s Orbit (cf. Ex. 4.1-3, 4.5, 4.8+9 in the textbook) 

Consider a Pluto-like planet \( m_P = 3 \times 10^{-6} M_{\odot} \) in its 2-D motion around the Sun (assumed to be fixed) using Newton’s universal law of gravitation, \( F_G = 4\pi^2 m_P/r^2 \), and Newton’s 2. law of motion, in astronomical base units (1AU, 1y, \( M_{\odot} \)). Use an orbit eccentricity of 0.3 (“true” value 0.248) and a semimajor axis of 40 AU (“true” value 39.5 AU).

(a) With the Sun to be located in a focal point of the elliptic orbit, use Kepler’s laws and the elliptic orbit formula to calculate analytically the perihelion, aphelion, the 2 pertinent speeds and the period of Pluto’s motion.

(b) Construct a FORTRAN code using the Euler-Cromer algorithm to describe Pluto’s motion assuming the Sun to be fixed (use a time-step width no larger than 0.1% of the presumed period). Choose your initial condition for Pluto on the positive x-axis with the velocity pointing in positive y direction, and the Sun at the origin. Attach the code and plots of the trajectory in the x-y-plane. Verify the period, semimajor and -minor axes and the Sun’s location in a focal point.

(c) Plot the kinetic, potential and total energy, as well as angular momentum, \( L \), as a function of time over a few periods. Compute \( L \) from its definition via a vector product in euclidean coordinates, \( \vec{L} = \vec{r} \times \vec{p} \).

(d) Replace the Euler-Cromer method with the Euler method, with initial conditions as in part (a). Plot and comment on the results. Does decreasing the time-step width help?

(e) In part (b), include Neptune’s motion (\( M_{Ne} = 5 \times 10^{-5} M_{\odot} \)) via a circular orbit of radius 30 AU starting on the negative x-axis (you can neglect the mutual force between Neptune and Pluto in the computation of the orbits). Over a time span of 100000 years, what is the distance of closest approach between the 2 planets and the corresponding (would-be) percentage distortion that Neptune exerts on Pluto (relative to the Sun’s force)?

(f) Modify the inverse-square force law by changing the exponent from -2 to -2.05 and plot Pluto’s orbit over many periods using the same initial conditions as in (b); comment on your results (e.g. precession period). Do another run with a slightly different initial condition (e.g. increasing \( v_0 \) by 0.1%) and plot the distance \( s(t) = |\vec{r}_1(t) - \vec{r}_2(t)| \) between the 2 trajectories; interpret your finding.