Homework Assignment #4

(Due Date: Friday, Feb. 26, 10:20 am, in class)

- 4.1 Pluto's Orbit (cf. Ex. 4.1-3, 4.5, 4.8+9 in the textbook) (1+3+1+1+2+2 pts.)Consider a Pluto-like planet $(m_P = 3 \cdot 10^{-6} M_{\odot})$ in its 2-D motion around the Sun (assumed to be fixed) using Newton's universal law of gravitation, $F_G = 4\pi^2 m_P/r^2$, and Newton's 2. law of motion, in astronomical base units $(1AU, 1y, M_{\odot})$. Use an orbit eccentricity of 0.3 ("true" value 0.248) and a semimajor axis of 40 AU ("true" value 39.5 AU).
 - (a) With the Sun to be located in a focal point of the elliptic orbit, use Kepler's laws and the elliptic orbit formula to calculate analytically the perihelion, aphelion, the 2 pertinent speeds and the period of Pluto's motion.
 - (b) Construct a FORTRAN code using the Euler-Cromer algorithm to describe Pluto's motion assuming the Sun to be fixed (use a time-step width no larger than 0.1% of the presumed period). Choose your initial condition for Pluto on the positive x-axis with the velocity pointing in positive y direction, and the Sun at the origin. Attach the code and plots of the trajectory in the x-y-plane. Verify the period, semimajor and -minor axes and the Sun's location in a focal point.
 - (c) Plot the kinetic, potential and total energy, as well as angular momentum, L, as a function of time over a few periods. Compute L from its definition via a vector product in euclidean coordinates, $\vec{L} \equiv \vec{r} \times \vec{p}$.
 - (d) Replace the Euler-Cromer method with the Euler method, with initial conditions as in part (a). Plot and comment on the results. Does decreasing the time-step width help?
 - (e) In part (b), include Neptune's motion $(M_{Ne} = 5 \cdot 10^{-5} M_{\odot})$ via a circular orbit of radius 30 AU starting on the negative x-axis (you can neglect the mutual force between Neptune and Pluto in the computation of the orbits). Over a time span of 100000 years, what is the distance of closest approach between the 2 planets and the corresponding (would-be) percentage distortion that Neptune exerts on Pluto (relative to the Sun's force)?
 - (f) Modify the inverse-square force law by changing the exponent from -2 to -2.05 and plot Pluto's orbit over many periods using the same initial conditions as in (b); comment on your results (e.g. precession period). Do another run with a slightly different initial condition (e.g. increasing v_0 by 0.1%) and plot the distance $s(t) = |\vec{r}_1(t) \vec{r}_2(t)|$ between the 2 trajectories; interpret your finding.