Name: PHY401 (Spring 2010), 02/08/10

Homework Assignment #3

(*Due Date*: Wednesday, February 17, 10:20 am, in class)

3.1 *Driven Harmonic Motion with Damping* (cf. Ex. 3.7 in the textbook) *(1+2+2+1 pts.)*
Consider the linear, damped, driven pendulum, defined by the differential equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\,\theta - 2\gamma\frac{d\theta}{dt} + \alpha_D \sin(\Omega_D t) \ . \tag{1}$$

(use g=9.8 m/s^2, l=9.8 m, γ=0.25 s^{-1}).

(a) Calculate analytically at what (approximate) value of Ω_D the resonance occurs. Do you expect the small-angle (linear) approximation to be good?

(b) Write a FORTRAN program to numerically calculate $\theta(t)$ using the Euler-Cromer Method. Plot $\theta(t)$ and $\omega(t) = d\theta/dt$, as well as the external driving acceleration, $\alpha_D(t)$, over a sufficiently long time to reach the steady-state solution. From the latter, extract the amplitude $\Theta_0(\Omega_D)$ and phase shift $\phi(\Omega_D)$ for at least 10 different driving frequencies mapping out the resonance structure, and plot $\Theta_0(\Omega_D)^2$ and $\phi(\Omega_D)$. Numerically extract the full-width at half maximum (FWHM) of the resonance curve and compare it to γ.

(c) For a driving frequency close to resonance, compute potential, kinetic and total energies, and plot them in the same graph over ca. 10 periods.

(d) Switch on nonlinear effects by replacing θ with $\sin\theta$ in the restoring force and plot+compare to your previous results for $\theta(t)$ and $\omega(t)$ using Ω_D close to resonance.

3.2 *Driven Pendulum and Transition/Approach to Chaos* (cf. Ex. 3.18 in the textbook) *(2+2 pts.)*
Consider the damped, driven, nonlinear pendulum of part (d) above for fixed Ω_D=$\frac{2}{3}s^{-1}$ but now varying the driving acceleration (force) α_D.

(a) Compute $|\Delta\theta(t)|$ for several trajectories with slightly different initial angle ($\Delta\theta_{\rm in} = 0.001\,rad$ or so) using α_D=1.2 rad/s^2. Plot the results and estimate the Lyapunov exponent λ of the system.

(b) Extend your code to compute (and then plot) the Poincaré sections of the motion for α_D=1.4, 1.44, 1.465, 1.48 and 1.485 rad/s^2.