# Homework Assignment \#2 

(Due Date: Friday, February 05, 10:20 am, in class)

### 2.1 Medieval Castle Defense

$$
(1+3+3+3+1 \text { pts. })
$$

A castle is built on the hills of a river valley, 80 m above the river/valley level. The castle's horizontal distance (from the bottom of the hill) to the nearside river bank is 110 m , and the river is another 70 m wide. The castle is equipped with several cannons which can eject solid smooth rock spheres of 30 cm diameter at a speed of $60 \mathrm{~m} / \mathrm{s}$ (the rock's mass density is $3000 \mathrm{~kg} / \mathrm{m}^{3}$ ). The knights are about to use the cannons in view of 500 enemy troops approaching the far-side river bank, but they have to figure out the appropriate launch angle. To save precious gunpowder, the court jester suggests to perform calculational estimates prior to the first shot.
(a) Neglecting air drag, calculate analytically the launch angle (and corresponding value) for the maximal range, the launch angle(s) required to hit the far-side of the river shore, and the corresponding sppeds upon impact.
(b) Write a FORTRAN code to compute the trajectory including a quadratic (in speed) air drag with a drag coefficient of 0.5 for the cannon ball (air density $\rho_{\text {air }}=1.29 \mathrm{~kg} / \mathrm{m}^{3}$ ). Can the cannon ball reach the far-side river shore? Attach the source code and a plot of trajectories including the one with the maximal range. Compute the speed upon impact.
(c) Determine the angle(s) in still air which allow(s) to hit a (pointlike) target on the nearside shore (attach a plot of these trajectories). Suppose a target has a size of $2 m$ length, which angle provides the largest accuracy (i.e., the largest launch-angle range, $\Delta \Theta_{0}$, within $\pm 1 m$ hit-range)? Use a linear interpolation technique to compute the hit locations (also in part (b)).
(d) Suddenly a horizontal head wind of 20 mph starts to blow, bringing also heavy rain fall; repeat part (c) for these conditions (approximate the effect of the rain drops by a $10 \%$ increase of the air density).
(e) BONUS QUESTION: estimate analytically the total vertical momentum which the rain drops (radius 0.2 cm ) impart on the cannon ball over the duration of its flight (use a vertical rain luminosity corresponding to 1 inch of precipitation in 5 minutes, in connection with an estimated terminal velocity of the falling drops; $\rho_{\text {water }}=1000 \mathrm{~kg} / \mathrm{m}^{3}$ ). Do you expect this effect to significantly affect the cannon-ball's trajectory?

