Name: PHY401 (Spring 2010), 01/20/10

Homework Assignment #1

(*Due Date*: Wednesday, January 27, 10:20 am, in class)

1.1 *Carbon Dating* *(1+6+3 pts.)*

The ${}^{14}_{6}C$ isotope is widely used for dating of ancient (organic-based) artifacts. It undergoes β^- decay with a half-life of $T_{1/2} = 5700\,years$. Suppose an ancient artifact originally contained $10^{-12}\,kg$ of ${}^{14}_{6}C$ (recall that $1\,mol$ of a substance, which has a mass corresponding to the atomic mass-number in $grams$, contains N_A=$6.022 \cdot 10^{23}$ particles).

(a) Derive analytically the relation between half-life, $T_{1/2}$, and lifetime, $\tau \equiv 1/\Gamma$, as defined in class.

(b) Write a FORTRAN code to numerically calculate the activity of the sample, defined as $R(t) = -dN/dt$, over a duration of 20000 $years$. Use a numerical time-step width of 100 $years$. Plot the result in appropriate units using GNU-PLOT together with the exact (analytical) solution in the same graph. Attach both the plot and your fortran source code.

(c) Increase the time-step width to 1000 $years$ and replot. Is the accuracy of the numerical solution still acceptable? (e.g., what is the percentage deviation from the exact result after 2 half-lifes?). Is the deviation from the exact result as large as you would expect from the neglected second-order term?