Name: PHY401 (Fall 2006), 11/08/06

Last 4 digits of UIN: Score:

Homework Assignment #8

(*Due Date*: Wednesday, November 22, 12:40 pm, in class)

8.1 *2-D Ising Model* (cf. Exercises 8.1, 8.2, 8.3 and 8.8 in the textbook) *(3+5+4+3 pts.)* Consider the 2-D Ising Model with nearest-neighbor interaction strength J=2 and at zero external magnetic field (except for part (d)).

(a) Use the Newton-Raphson root finder algorithm to numerically solve the mean-field selfconsistency relation for the average magnetization,

$$\langle s \rangle = \tanh\left(\frac{zJ}{k_B T}\langle s \rangle\right) \ , \tag{1}$$

in the temperature range from zero to just above T_c. Compare your numerical results at large and small(!) temperature to analytic results obtained from a suitable Taylor expansion.

(b) Write a FORTRAN program to numerically study the 2-D Ising Model on a $n \times n$ lattice with periodic boundary conditions (N: total number of spins, $M = N\langle s \rangle$). Calculate the magnetization as a function of temperature (allowing for enough Monte Carlo sweeps to reach equilibrium) and determine the critical temperature as well as the critical exponent β in $M \propto (T_c - T)^\beta$.
(*Hints*: to extract β, concentrate on the temperature region $T = (0.9 - 1)T_c$. For several values of T, plot $\log(M)$ vs. $\log(T_c - T)$ and take as a best estimate the case where a straight line appears most appropriate; perform this procedure for two lattice sizes, n=15 and 25, to check for finite-size effects.)

(c) Calculate the specific heat per spin, C/N, for 5 different lattice sizes between n=5 and 40, using the fluctuation-dissipation theorem, $C = (\Delta E)^2/(k_B T^2)$, and verify the approximate "finite-size scaling" relation $C_{max}/N \propto \log(n)$.
(*Hint*: make sure to use sufficient temperature resolution when determining the maximum in C/N as you increase n.)

(d) Introduce an external magnetic field, H, into your code and calculate (and plot) the dependence of the magnetization on H for a fixed temperature, $T = T_c/4$. Use fine enough steps in H to convince yourself that the transition in $M(H)$ is of first order. Verify the hysteresis effect by varying H across the transition both from negative to positive and from positive to negative.