Name: PHY401 (Fall 2006), 10/25/06

Last 4 digits of UIN: Score:

# Homework Assignment #7

(*Due Date*: Wednesday, November 08, 12:40 pm, in class)

7.1 *3-D Self-Avoiding Walk* (cf. Exercise 7.6 in the textbook) *(4 pts.)*
Write a FORTRAN program to simulate a self-avoiding random walk in 3 dimensions, taking steps of unit length in $x$-, $y$- or $z$-direction. In particular, make sure that each walk of step-length $n$ (polymer with given molecule no. $n$) has the same probability. Show that the motion (polymer size) is diffusive with $\sqrt{\langle r^2 \rangle} = A\,t^\nu$; determine the value of the Flory exponent $\nu$ (you can use either the statistical "trial" method or the exact depth-first enumeration method for your computation).

7.2 *Diffusion Limited Aggregation and Fractal Dimension* (cf. Ex. 7.19) *(5 pts.)*

(a) Write a FORTRAN program to generate DLA clusters in 3 dimensions. To save computer time, implement the following features into your simulation: start from a seed at the origin, let random walkers start from a random position on a circle (but on your numerical grid) about 8-10 units away from the origin. When growing the cluster, keep the circle about 5 units away from the perimeter of the cluster. If a walker has wandered off to more than 1.5 times the starting radius, terminate the walk and begin a new one. Grow several clusters to a size of several ten-thousands attachments.

(b) Calculate the dimensionality of the clusters you have generated in part (a) by finding the exponent, $d_f$, in the mass radius relationship, $m(r) \propto r^{d_f}$, and evaluate the average $d_f$ as your best estimate for the dimensionality of the DLA cluster.

7.3 *2-D Percolation and Critical Exponent* (cf. Figs. 7.26, 7.27, Ex. 7.31) *(6 pts.)*

(a) Write a FORTRAN program to simulate the percolation transition on a $N \times N$ lattice by subsequently (randomly) filling lattice sites corresponding to an increase in occupation probability, $p$. Determine the critical probability $p_c$ by checking, after each new entry, for the first appearance of a spanning cluster, and plot the spanning cluster (cf. Fig. 7.28 in the textbook). Repeat this procedure for various lattice sizes $N$ (e.g. $N$=10 to 40 in steps of 5) and plot $p_c(N^{-1})$ to extrapolate to the infinite-size limit, $p_c(0)$.

(b) Starting from a single "seed" site, use the breadth-first growth algorithm to generate a percolating (2-D) cluster for various site occupation probabilities $p < p_c = 0.593$ (the growth should always terminate itself). For each $p$, estimate the "correlation length", $\xi(p)$, by computing the average distance of all cluster sites from the initial seed site. Fit $\xi(p)$ with a power law $\xi \propto |p - p_c|^{-\nu}$ in order to estimate the critical exponent $\nu$.