Name: PHY401 (Fall 2006), 10/13/05

Last 4 digits of UIN: Score:

Homework Assignment #6

(*Due Date*: Monday, October 23, 12:40 pm, in class)

6.1 *Ideal Wave Equation and Spectral Analysis* (cf. Ex. 6.1, 6.9) *(8 pts.)*
Write a FORTRAN program to numerically calculate the signal propagation on an ideal string (length $L = 1m$, tension force $F_T = 900N$, mass density $\mu = 0.010kg/m$) using the ideal wave equation.

(a) Assume one end of the string fixed and the other end open. Impart an initial Gaussian pluck on the string, $y(x, t_{0,1}) = \exp[(x - x_0)^2/2\sigma_x^2]$ with x_0 40% away from the open end, and a full width at half maximum of $0.05m$ (what is the corresponding value of σ_x?). Study the signal propagation by plotting time snapshots of your solution, $y(x, t_s)$, as the signal moves across the string. How are the signals reflected at the ends?

(b) Perform a Fourier analysis of the string excitation in part (a) (i.e., for a fixed position on the string, record $y(t)$ and evaluate the Fourier transform using a suitable time discretization) and plot the power spectrum vs. frequency. Interpret the peaks in the spectrum in terms of the harmonics of the string.

(c) Consider now the same string but with both ends fixed. Initialize it with a more realistic "triangular" pluck, with a maximum $y(L/3, t_{0,1}) = 0.2m$, which linearly connects to the ends (see Fig. 1 below). Study its time dependence and power spectrum in analogy to parts (a) and (b).

Fig.1:
Realistic Pluck

6.2 *Stiffness Term in Wave Equation* (cf. Ex. 6.15) *(2 pts.)*
The wave equation with stiffness correction for a string under tension F_T reads

$$\frac{\partial^2 y}{\partial t^2} = c^2 \left(\frac{\partial^2 y}{\partial x^2} - \frac{Y}{F_T} \frac{\partial^4 y}{\partial x^4} \right) , \tag{1}$$

with Y: Young's modulus (no numerical work required in this problem).

(a) Derive the symmetric finite difference expression of the 4. order derivative term suitable for numerical evaluation.

(b) Derive the finite difference expression for $y(i, n+1)$ for the string displacement y at time t_{n+1} and position x_i in terms of y at times $t_{n,n-1}$ and positions $x_{i-2,i-1,i,i+1,i+2}$.