Name: PHY401 (Fall 2006), 10/04/05

Last 4 digits of UIN: Score:

Homework Assignment #5

(*Due Date*: Friday, October 13, 12:40 pm, in class)

5.1 *Poisson Equation for Dipole* (cf. Ex. 5.7) *(6 pts.)*
Write a FORTRAN program to numerically calculate the potential of an electric dipole, i.e., two point charges $Q/\epsilon_0 = \pm 1$ separated by a distance $a = 0.5$. Solve the Poisson equation in Cartesian coordinates but impose a spherical boundary condition, $V(R) = 0$, at a large distance (e.g., $R = 5$).

(a) Starting from an initial condition of zero potential use the Jacobi relaxation algorithm with appropriate numerical tolerance and grid density to obtain the (converged) solution. Plot the equipotential lines. Compare your result for V(r) (r: distance from origin) to the expected large distance behavior of the dipole potential.

(b) Investigate how the number of required iteration steps, N_{iter}, increases with reducing the tolerance (error) limit, and plot it, i.e., $N_{iter}(tol)$.

(c) Write a second program by modifying the algorithm to using the Simultaneous Over Relaxation (SOR) method, and allowing for variable grid density. For fixed accuracy in the solution (not total tolerance!), investigate (and plot) how the number of iteration steps, N_{iter}, depends on the number n=n_x=n_y of grid points, showing that for the Jacobi and SOR method, $N_{iter} \propto n^2$ and $N_{iter} \propto n$, respectively.

5.2 *Helmholtz Coils* (cf. Ex. 5.7) *(4 pts.)*
Use Biot-Savart's Law for the magnetic field induced by a thin-wire current,

$$d\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \frac{d\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} , \tag{1}$$

to construct a FORTRAN code calculating the $\vec{B}$-field for an arrangement of 2 current loops (circular wires), each of radius r oriented parallel to the x-y plane and separated by a distance r (cf. also Fig. 5.18 in the textbook). Choose, e.g., $r = 1m$, $\mu_0 I$=10 in SI units running counter-clockwise. Calculate and plot the x-, y- and z-components of the magnetic field for
(a) $x = y = 0m$ as a function of z (check with the analytic result!)
(b) $y = z = 0$ as a function of x.