Name: PHY401 (Fall 2006), 09/25/06

Last 4 digits of UIN: Score:

Homework Assignment #4

(*Due Date*: Wednesday, October 04, 12:40 pm, in class)

4.1 *Halley's Comet* (cf. Ex. 4.4, 4.5 in the textbook) *(5 pts.)*
Consider elliptical orbits of planets around the Sun using Newton's universal law of gravitation, $F_G = 4\pi^2 M_P/r^2$, and Newton's 2. law of motion, in astronomical units ($1AU$, $1y$, $M_\odot$). Construct a FORTRAN code to describe the planetary motion assuming the Sun to be fixed in one of the focal points of the ellipse (*hint: use time steps no larger than 0.01*).

(a) Consider a "test planet" with initial condition $\vec{r}_{ini} = (1, 0)$, $\vec{v}_{ini} = (0, 4.5)$. Calculate the eccentricity of the orbit analytically and compare it to the value extracted from the numerically computed orbit. Over four periods, plot the kinetic, potential and total energy and verify that the latter is conserved.

(b) Use your code to construct the orbit of Halley's comet, which has a period $76\,y$ and a distance of closest approach to the sun of $0.59\,AU$. What is its eccentricity and maximal distance from the Sun? With which planets of the solar system could the comet possibly collide?

4.2 *Precession of Mercury's Orbit due to GR* (cf. Ex. 4.10 in the textbook) *(5 pts.)*
Based on the planetary motion code constructed above, evaluate the precession of Mercury's orbit due to general-relativistic effects according to the following steps:

(a) Extend your code by implementing the leading-order correction term derived from general relativity into Newton's gravitational force,

$$F_G \simeq \frac{4\pi^2 M_M}{r^2}(1 + \frac{\alpha}{r^2}) \ . \tag{1}$$

(b) Using $\alpha = 0.002$, record the precession angle θ_p of Mercury's semi-major axis over several orbital periods and plot the result as $\theta_p(t)$. Extract the slope of an (eye-ball) fit to the "data" to obtain the precession rate $\dot{\theta}_p$.

(c) Repeat part (b) for α=0.004, 0.003 and 0.001. Plot $\dot{\theta}_p(\alpha)$, extract the slope of an (eye-ball) fit to the "data" to determine $d\dot{\theta}_p/d\alpha$, and finally calculate the precession rate (in $arcsec/100y$) for the theoretical value of $\alpha = 1.1 \times 10^{-8} AU^2$. Is it consistent with the expected result?