Name: PHY401 (Fall 2006), 09/13/06

Last 4 digits of UIN: Score:

Homework Assignment #3

(*Due Date*: Friday, September 22, 12:40 pm, in class)

3.1 *Nonlinear Pendulum* (cf. Exercise 3.8 in the textbook) *(4 pts.)*

The nonlinear pendulum is defined by the differential equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(\theta)\ . \tag{1}$$

(use g=9.8 m/s^2, l=1 m).

(a) Write a FORTRAN program to numerically calculate $\theta(t)$ using the Euler-Cromer Method. Plot the result for the amplitudes $\theta_0 = \pi/4, 3\pi/4$. Numerically determine the dependence of the period on the amplitude and plot it for $0 < \theta_0 < \pi$ (using reasonably fine steps in θ_0). Give a qualitative interpretation of the result.

(b) Verify energy conservation by calculating total, kinetic and potential energy as a function of time and plot them over 5 periods for $\theta_0 = 3\pi/4$.

3.2 *Pendulum Motion and Chaos* (cf. Ex. 3.13, 3.20 in the textbook) *(6 pts.)*

Consider a damped, driven, nonlinear pendulum defined by the differential equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(\theta) - 2\gamma\frac{d\theta}{dt} + \alpha_D\sin(\Omega_D t). \tag{2}$$

(use g=9.8 m/s^2, l=9.8 m, γ=0.25/s, Ω_D=$\frac{2}{3}$ rad/s).

(a) Write a FORTRAN program to calculate $|\Delta\theta(t)|$ for several trajectories with slightly different initial angle (0.001 rad or so) using α_D=1.2 rad/s^2. Plot the results and estimate the Lyapunov exponent λ of the system.

(b) For α_D=1.35-1.5 rad/s^2, calculate and plot the Bifurcation diagram of the pendulum; magnify (and scan more carefully) the regime α_D>1.465 rad/s^2 and obtain a numerical estimate of the Feigenbaum-δ of the system.