Name: PHY401 (Fall 2005), 10/12/05

Homework Assignment #5

(*Due Date*: Friday, October 21, 12:40 pm, in class)

4.1 *Laplace Boundary Problem* (cf. Ex. 5.4 and 5.7 in the textbook) *(6 pts.)*
Write a FORTRAN program to calculate the potential and electric field of the "finite capacitor" (as shown in the figure below; cf. also Fig. 5.6 in the textbook).

(a) Starting from an initial condition of zero potential in the inner region, use the Jacobi relaxation algorithm with appropriate numerical tolerance and grid density to obtain the (converged) solution. Plot the solutions for the potential, as well as x- and y-components of the electric field, in 1-D sections for y=0, ±2 and x=0, ±2 as a function of x and y, respectively.

(b) Investigate how the number of required iteration steps, N_{iter}, increases with reducing the tolerance (error) limit, and plot it, i.e., $N_{iter}(tol)$.

(c) Write a second program by modifying the algorithm to using the Simultaneous-Over-Relaxation (SOR) method, and allowing for variable grid density. For fixed accuracy in the solution (not total tolerance!), investigate (and plot) how the number of iteration steps, N_{iter}, depends on the number n=n_x=n_y of grid points, showing that for the Jacobi and SOR method, $N_{iter} \propto n^2$ and $N_{iter} \propto n$, respectively.

Figure:

4.2 *Biot-Savart Law* *(4 pts.)*
Use Biot-Savart's Law for the magnetic field induced by a thin-wire current,

$$d\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \frac{d\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} , \qquad (1)$$

to construct a FORTRAN code calculating the $\vec{B}$-field for a quadratic current loop (side length $2m$, $\mu_0 I$=10 in SI units, running counter-clockwise) lying (centered) in the x-y plane. Calculate and plot the x-, y- and z-components of the magnetic field for
(i) z=$1m$, y=0 as a function of x
(ii) y=0, x=$0.5m$ as a function of z.