Name: PHY401 (Fall 2005), 10/02/05

Homework Assignment #4

(*Due Date*: Wednesday, October 12, 12:40 pm, in class)

4.1 *Kepler's 3.Law of Planetary Motion* (cf. Ex. 4.5 and 4.8 in the textbook) *(7 pts.)*
Consider elliptical orbits of planets around the Sun using Newton's universal law of gravitation, $F_G = 4\pi^2 M_P/r^2$, and Newton's 2. law of motion, using astronomical units ($1AU$, $1y$, $M_\odot$). Construct a FORTRAN code to describe the planetary motion assuming the Sun to be fixed in one of the focal points of the ellipse (*hint: use time steps no larger than 0.01*).

(a) Consider a "test planet" with initial condition $\vec{r}_{ini} = (1, 0)$, $\vec{v}_{ini} = (0, 5)$. Calculate the eccentricity of the orbit analytically and compare it to the value extracted from the numerically computed orbit. Over three periods, plot the kinetic, potential and total energy and verify that the latter is conserved.

(b) Verify Kepler's 3. law by explicitly recording the period (for several revolutions) for the following planet orbits (using and quoting accurate initial conditions):
(i) Mercury (semi-major axis a=0.39, eccentricity e=0.206);
(ii) Earth (a=1.00, e=0.017);
(iii) Saturn (a=9.54, e=0.056);
(iv) Pluto (a=39.53, e=0.248).

4.2 *Precession of Mercury's Orbit due to GR* (cf. Ex. 4.10 in the textbook) *(3 pts.)*
Based on the planetary motion code constructed above, evaluate the precession of Mercury's orbit due to general-relativistic effects according to the following steps:

(a) Extend your code by implementing the leading-order correction term derived from general relativity into Newton's gravitational force,

$$F_G \simeq \frac{4\pi^2 M_M}{r^2}\left(1 + \frac{\alpha}{r^2}\right) . \tag{1}$$

(b) Using $\alpha = 0.005$, record the precession angle θ_p of Mercury's semi-major axis over several orbital periods and plot the result as $\theta_p(t)$. Extract the slope of an (eye-ball) fit to the "data" to obtain the precession rate $\dot{\theta}_p$.

(c) Repeat part (b) for α=0.002, 0.001 and 0.0005. Plot $\dot{\theta}_p(\alpha)$, extract the slope of an (eye-ball) fit to the "data" to determine $d\dot{\theta}_p/d\alpha$, and finally calculate the precession rate (in $arcsec/100y$) for the theoretical value of $\alpha = 1.1\times10^{-8} AU^2$. Is it consistent with the expected result?