Name: PHY401 (Fall 2005), 09/19/05

Homework Assignment #3

(*Due Date*: Wednesday, September 28, 12:40 pm, in class)

3.1 *Simple Harmonic Motion* (cf. Exercise 3.1 in the textbook) *(2 pts.)*
The simple pendulum is defined by the differential equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \ . \tag{1}$$

(use g=9.8 m/s^2, l=1 m).

(a) Write a FORTRAN program to numerically describe the corresponding motion of a simple pendulum using the Euler-Cromer Method. Plot the result and compare to the exact solution.

(b) Verify energy conservation by calculating total, kinetic and potential energy as a function of time and plot over 5 periods.

3.2 *Pendulum Motion and Chaos* (cf. Ex. 3.13, 3.14 in the textbook) *(8 pts.)*
Consider a damped, driven, nonlinear pendulum defined by the differential equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(\theta) - 2\gamma\frac{d\theta}{dt} \ + \alpha_D \sin(\Omega_D t). \tag{2}$$

(use g=9.8 m/s^2, l=9.8 m, γ=0.25/s, Ω_D=$\frac{2}{3}$ rad/s, α_D=1.2 rad/s^2).

(a) Write a FORTRAN program to calculate $|\Delta\theta(t)|$ for several trajectories with slightly different initial angle (0.001 rad or so). Plot the results and estimate the Lyapunov exponent λ of the system.

(b) Investigate the change of λ under (moderate) variations of γ.

(c) Calculate the Poincaré section in phase space, $\omega(\theta)$, for the system in part (a) for times $t = n(2\pi)/\Omega_D$ (with integer n=1, 2, ...). Use a sufficiently long running time to map out and plot the strange attractors.