Electromagnetic emission from the CGC at early stages of heavy ion collisions

François Gelis

CEA / DSM / SPhT
Outline

- Overview of heavy ion collisions
- Sources of photons
- Introduction to the Color Glass Condensate
- Photons in pA collisions
- Photons in AA collisions (prospective, not much done yet...)
- Conclusions and perspectives

Based on:

Baltz, FG, McLerran, Peshier, nucl-th/0101024
FG, Kajantie, Lappi, hep-ph/0409048
Heavy Ion collisions

- \( \tau \sim 0 \text{ fm/c} \)
- Production of hard particles
- calculable with the tools of perturbative QCD
- $\tau \sim 0.2$ fm/c
- Production of semi-hard particles
- relatively small momentum: $p_\perp \lesssim 1$–2 GeV
- make up for most of the multiplicity
- sensitive to the physics of saturation (CGC)
Thermalization

- experiments suggest a fast thermalization
- but this is still not fully understood from QCD
Heavy ion collisions

- Quark gluon plasma
Hot hadron gas
Chemical freeze-out :
density too small to have inelastic interactions

Kinetic freeze-out :
no more elastic interactions
Sources of photons

- Prompt photons
- when produced at forward rapidities and moderate $p_\perp$, may be affected by the large density of gluons at small $x$
  - higher twist corrections, calculable in the CGC framework, should be included
Sources of photons

- Thermal photons
Sources of photons

- Decay photons
Sources of photons

- Medium induced jet fragmentation
Sources of photons

- **Photons from the CGC**
- Processes that have only gluons in the initial state
- Calculable in the CGC framework as long as the classical description holds
  - breaks down when the gluon phase-space density is of order unity or smaller
Where does the CGC stand?

- describes the content of nucleons and nuclei at small $x$
- framework to calculate the production of semi-hard particles
- provides initial conditions for the subsequent evolution
Nucleon at rest

- Very complicated non-perturbative object...
- Contains lots of fluctuations at all spacetime scales smaller than $\Lambda_{QCD}^{-1}$
- Only the fluctuations that live longer than the external probe are relevant in the interaction process
- All the effect of the shorter-lived fluctuations is to renormalize the coupling and masses
- Interaction processes are very complicated if the nucleon constituents have non-trivial interactions over the characteristic timescale seen by the probe
Nucleon at high energy

- **Time dilation** of all the internal timescales of the nucleon
- The interactions among the constituents now occur over timescales much larger than the interaction with the external probe, the constituents behave as if they were free
- Some of the fluctuations now live long enough to be seen. The nucleon appears denser at high energy. The new partons have a smaller momentum fraction \( x \)
- Previous fluctuations are now frozen over the timescale of the probe, and merely act as static sources of new partons
Parton model

- At the time of the interaction, the nucleon can be seen as a collection of **free constituents**, called **partons**

- It can be described by **non-perturbative parton distributions** that depend on the momentum fraction \( x \) of the partons

- One simply has to calculate the **perturbative cross-section** between the probe and the partons. If the density of partons is low, only one parton will interact with the probe

- One can separate the perturbative hard scattering from the non-perturbative distribution functions, because the strong interactions that are responsible for these non-perturbative aspects occur on much larger timescales (**factorization**)

- Parton distributions depend also on a “transverse resolution scale”, \( Q \), because of collinear splitting:

\[
Q^{-1}
\]
at low energy, only valence quarks are present in the hadron wave function
when energy increases, new partons are emitted

the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln\left(\frac{1}{x}\right)$, with $x$ the longitudinal momentum fraction of the gluon

at small-$x$ (i.e. high energy), these logs need to be resummed
as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL) Kuraev, Lipatov, Fadin (1977), Balitsky, Lipatov (1978)
evolution and saturation

- eventually, the partons start overlapping in phase-space
- parton recombination becomes favorable
- after this point, the evolution is non-linear:
  the number of partons created at a given step depends non-linearly
  on the number of partons present previously

Iancu, Leonidov, McLerran (2001)
Saturation criterion


- Number of partons per unit area:

\[ \rho \sim \frac{xG(x, Q^2)}{\pi R^2} \]

- Recombination cross-section:

\[ \sigma_{gg \rightarrow g} \sim \frac{\alpha_s}{Q^2} \]

- Recombination if \( \rho \sigma_{gg \rightarrow g} \gtrsim 1 \), or \( Q^2 \lesssim Q_s^2 \), with:

\[ Q_s^2 \sim \frac{\alpha_s xG(x, Q_s^2)}{\pi R^2} \sim A^{1/3} \frac{1}{x^{0.3}} \]

- At saturation, the gluon phase-space density is:

\[ \frac{dN_g}{d^2 \vec{x}_\perp d^2 \vec{p}_\perp} \sim \frac{\rho}{Q^2} \sim \frac{1}{\alpha_s} \]
Boundary defined by \( Q^2 = Q_s^2(x) \)
McLerran, Venugopalan (1994)
Iancu, Leonidov, McLerran (2001)

- Small-\(x\) modes have a large occupation number
  - they are described by a classical color field \(A^\mu\)

- The large-\(x\) modes, slowed down by time dilation, are described as frozen color sources \(\rho_\alpha\). They act as sources for the modes at lower values of \(x\)

- The classical field obeys Yang-Mills’s equation:

\[
[D_\nu, F^{\nu\mu}]_\alpha = \delta^\mu_+ \delta(x^-) \rho_\alpha(\vec{x}_\perp)
\]

- The color sources \(\rho_\alpha\) are random, and described by a distribution functional \(W_{x_0}[\rho]\), with \(x_0\) the separation between “small-\(x\)” and “large-\(x\)”.
The solution of the classical Yang-Mills equation is the sum of all the **tree diagrams** that connect the point where the gauge field is evaluated to an arbitrary number of sources:

![Diagram](image)

> this solution incorporates all the diagrams responsible for saturation
Quantum evolution

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

- The distribution $W_{x_0}[\rho]$ evolves with $x_0$ (more modes are included in $W$ as $x_0$ decreases)

- In a high density environment, the newly created gluons can interact with all the sources already present:

- The evolution is governed by the JIMWLK equation:

$$\frac{\partial W_{x_0}[\rho]}{\partial \ln (1/x_0)} = \frac{1}{2} \int_{\vec{x}_\perp, \vec{y}_\perp} \frac{\delta}{\delta \rho_a(\vec{x}_\perp)} \chi_{ab}(\vec{x}_\perp, \vec{y}_\perp) \frac{\delta}{\delta \rho_b(\vec{y}_\perp)} W_{x_0}[\rho]$$

- $\chi_{ab}$ depends on $\rho$ to all orders
- reduces to BFKL in the low density regime
- diffusion equation in a functional space
Initial condition - MV model

- The JIMWLK equation must be completed by an initial condition, given at some moderate $x_0$
- As with DGLAP, the problem of finding the initial condition is in general non-perturbative
- The McLerran-Venugopalan model is often used as an initial condition at moderate $x_0$ for a large nucleus:

  - partons distributed randomly
  - many partons in a small tube
  - no correlations at different $\vec{x}_\perp$

The MV model assumes that the density of color charges $\rho(\vec{x}_\perp)$ has a Gaussian distribution:

$$W_{x_0}[\rho] = \exp \left[ - \int d^2 \vec{x}_\perp \frac{\rho_a(\vec{x}_\perp) \rho_a(\vec{x}_\perp)}{2 \mu^2(\vec{x}_\perp)} \right]$$
In the deeply saturated regime, one expects a non-local gaussian distribution of color sources:

\[
W_x[\rho] = \exp \left[ -\int_{\vec{x}_\perp, \vec{y}_\perp} \rho_a(\vec{x}_\perp) \rho_a(\vec{y}_\perp) \frac{\rho(x, \vec{x}_\perp - \vec{y}_\perp)}{2\mu^2(x, \vec{x}_\perp - \vec{y}_\perp)} \right]
\]

with:

\[
\mu^2(x, \vec{k}_\perp) = \frac{2}{\gamma c} k^2_\perp \ln \left( 1 + \left( \frac{Q_s^2(x)}{k^2_\perp} \right)^\gamma \right)
\]

- the color correlation length is now \(Q_s^{-1} \ll 1 \text{ fm}\)
- this regime has geometrical scaling:
  dimensionless functions of \(x\) and \(k_\perp\) depend only on the combination \(k_\perp/Q_s(x)\)
Calculation of observables

- Observables are first calculated in the classical field created by one configuration of the sources \( \rho \), and then averaged over the hard sources \( \rho \):

\[
O = \int D\rho \ W_{x_0}[\rho] \ O[\rho]
\]

**Note:** this average restores gauge invariance

- Typically, the observable \( O \) is the square of some transition amplitude \( O[\rho] = |M|^2 \), with \( M = \langle \alpha_{\text{in}} | \beta_{\text{out}} \rangle \)

- The amplitude \( M \) must be calculated in the presence of the background field \( A^\mu[\rho] \)

- \( \langle \langle \alpha_{\text{in}} | \beta_{\text{out}} \rangle \rangle^2 \rho \) is the probability that the final state contains the particles \( \beta \), plus any number of fragments of the projectiles (i.e. an inclusive probability)
Leading twist shadowing

- Interactions between the partons of the target:

> At small $x$, the wave function of a parton “spreads” outside of the nucleon it belongs to, so that it can interact with partons from other nucleons. Gluon recombination implies:

$$xG_{\text{nucleus}}(x, Q^2) < A xG_{\text{nucleon}}(x, Q^2)$$

> At small $x$, one has a suppression of cross-sections:

$$d\sigma_{pA}/d^2\mathbf{p}_\perp \sim A^\alpha \quad \text{with} \quad \alpha < 1$$

> These interactions are the same as those involved in saturation
Multiple scatterings

- Because of the large parton density at small $x$ in the target, an external probe can interact several times:

- One of the scatterings “produces” the final state, and the others merely change its momentum (“higher twist” shadowing)

- Each additional scattering brings a correction $\alpha_s A^{1/3} \mu^2 / p_\perp^2$
  - Important effect at small $p_\perp$, despite the $\alpha_s$ suppression

- At leading order, multiple scattering only affect the momentum distribution of the final particles, but not their total number. The suppression at small $p_\perp$ is compensated by an increase at larger $p_\perp$ (Cronin effect)
Multiple scatterings

- At high $p_\perp$, a single scattering dominates:

  - Standard result for a random walk in an external potential, when the potential does not decrease fast at large momentum ("intermittency")
  - Differential cross-sections scale like the atomic number $A$ at high $p_\perp$, one recovers LO pQCD

- The MV model describes correctly multiple scatterings, but does not contain any leading twist shadowing. The latter arises from the evolution to smaller $x$'s with JIMWLK
Inclusive photon spectrum in pQCD

\[ \mathcal{O}(\alpha_s^0) : \]

(Drell-Yan: only for virtual photons)
Inclusive photon spectrum in pQCD

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(Drell-Yan: only for virtual photons)
Inclusive photon spectrum in pQCD

\[ \mathcal{O}(\alpha_s^0) : \]

(Drell-Yan: only for virtual photons)

\[ \mathcal{O}(\alpha_s) : \]

Diagrams that are of \( \mathcal{O}(\alpha_s^2) \), but enhanced by large logs:

\[ \cdots \]
The fragmentation function $D_{q \to \gamma}$ behaves as:

$$D_{q \to \gamma}(z, p_\perp^2) \sim \ln \left( \frac{p_\perp^2}{\Lambda_{QCD}} \right) \sim \frac{1}{\alpha_s(p_\perp^2)}$$

the log comes from the integration of the phase-space of the quark produced along with the photon

- the fragmentation contribution can be eliminated to a large extent by isolation cuts around the photon

It obeys a non-homogeneous DGLAP equation:

$$\frac{\partial D_{q \to \gamma}}{\partial \ln(p_\perp^2)} = K_{q \to \gamma} + \int_z^1 \frac{dx}{x} \left[ P_{qq} \left( \frac{z}{x} \right) D_{q \to \gamma}(x) + P_{qg} \left( \frac{z}{x} \right) D_{g \to \gamma}(x) \right]$$

- this equation resums all corrections due to collinear gluon emission, of order $[\alpha_s \ln(p_\perp^2)]^n$
- $K_{q \to \gamma}$ is the log derivative of the direct $q \to q\gamma$ splitting process
Nuclear modifications

- At small $x$, processes that have a gluon in the initial state are enhanced compared to those that don’t

- Multi-gluon (“higher twist”) contributions:
  - no longer suppressed if the gluon density is large
  - the easiest way to calculate them is to replace the projectile probed at small $x$ by its classical color field:

  $$A^\mu(x) \sim \mathcal{O}(g^{-1}) \quad \Rightarrow \quad gA^\mu(x) \sim \mathcal{O}(1)$$

- $n$-gluon terms are as important as the single gluon one
Nuclear modifications

- In the previous diagrams, one of the projectiles is described in terms of its classical field, while the other is described by ordinary quark distributions.
- This is a valid description at forward rapidities, if $x_1 \ll x_2$.
- The previous contributions are the dominant ones. Diagrams like

\[ \begin{array}{c}
\begin{array}{c}
\includegraphics[width=0.5\textwidth]{diagram.png}
\end{array}
\end{array} \]

are suppressed by inverse powers of the collision energy, because they require the photon to be produced inside the Lorentz contracted target.
- A more systematic way to obtain these leading contributions is to compute the amplitude $\langle q_{\text{in}} | q_{\gamma_{\text{out}}} \rangle$ in the presence of the background color field of the other projectile.
Photon and dilepton cross-sections

- Relation between amplitudes and cross-sections:
  - Photon production:
    \[ 4\omega_k \omega_q \frac{d\sigma}{d^3k d^3\tilde{q}} = \frac{1}{64\pi^5 p} \delta(p^- - q^- - k^-) |\mathcal{M}|^2 \]
  - Dilepton production:
    \[ 2\omega_q \frac{d\sigma}{d^4k d^3\tilde{q}} = \frac{\alpha_{\text{em}}}{192\pi^6 p^- k^2} \delta(p^- - q^- - k^-) |\mathcal{M}|^2 \]

  with \( \langle q(\vec{p})_{\text{in}} | q(\vec{q}) \gamma(\vec{k})_{\text{out}} \rangle = 2\pi \delta(p^- - q^- - k^-) \mathcal{M} \)

- Classical color field in covariant gauge:
  \[ A_\mu^a(x) = \delta^\mu^+ \delta(x^-) \frac{1}{\nabla_\perp^2} \rho_a(\vec{x}_\perp) \]

  Note: no \( x^+ \) dependence
  ▶️ the \( - \) component of the momentum is conserved
Quark propagator in the classical color field:

\[
\sum_{n=1}^{\infty} \gamma_{\perp}^{-\perp} \int d^2 \vec{x}_\perp e^{i(\vec{q}_\perp - \vec{p}_\perp) \cdot \vec{x}_\perp} \left[ U^\epsilon(p^-) (\vec{x}_\perp) - 1 \right]
\]

with:

\[
U(\vec{x}_\perp) = \mathcal{P} \exp \left[ -ig \int_{-\infty}^{+\infty} dz^- A^+_{a} (z^-, \vec{x}_\perp) t^a \right]
\]

Photon production amplitude (for a massless quark):

\[
|\mathcal{M}|^2 = 8\pi R^2 C(\vec{q}_\perp + \vec{k}_\perp - \vec{p}_\perp) \left\{ (p^-)^2 + (q^-)^2 \left[ \frac{2p \cdot q}{D_p D_q} - \frac{1}{D_p} - \frac{1}{D_q} \right] \right.

+ k^2 \left[ \frac{p^-^2}{D_p^2} + \frac{q^-^2}{D_q^2} + \frac{(p^- + q^-)^2}{D_p D_q} \right] \right\}
\]

with:

\[
D_p = (p - k)^2, \quad D_q = (q + k)^2
\]
Photon and dilepton cross-sections

- “Gluon distribution” of the dense projectile:

\[ C(\vec{l}_\perp) = \frac{1}{N} \int d^2 \vec{x}_\perp e^{i \vec{l}_\perp \cdot \vec{x}_\perp} \text{tr} \left\langle U(0)U^\dagger(\vec{x}_\perp) \right\rangle_\rho \]

- this function controls how momentum flows from the dense projectile to the quark
- all the information about the dense projectile is encoded there

- Example: MV model for the distribution of sources:

\[ W_x[\rho] = \exp \left[ - \int d^2 \vec{x}_\perp \frac{\rho_\alpha(\vec{x}_\perp)\rho_\alpha(\vec{x}_\perp)}{2\mu^2(x)} \right] \]

\[ C(\vec{l}_\perp) = \int d^2 \vec{x}_\perp e^{i \vec{l}_\perp \cdot \vec{x}_\perp} e^{\frac{1}{4} \alpha_s C_f \mu^2 x_\perp^2 \ln(1/x_\perp^2 \Lambda^2)} \]

- \( \Lambda \) is an infrared cutoff required to enforce color neutrality at the scale of nucleon size
- at large momentum transfer: \[ C(\vec{l}_\perp) \approx g^2 \mu^2 C_f / l_\perp^4 \], which comes from single gluon exchange
Connection with Deep Inelastic Scattering:

\[ F_2 \sim \sigma_{\gamma^* p} = \int_0^1 dz \int d^2 \vec{x}_\perp |\psi(z, x_\perp; Q^2)|^2 \sigma_{\text{dipole}}(x, x_\perp) \]

with:

\[ \sigma_{\text{dipole}}(x, x_\perp) = 2\pi R^2 \text{tr} \left[ 1 - \frac{1}{N} \text{tr} \left\langle U(0)U^\dagger(\vec{x}_\perp) \right\rangle_\rho \right] \]

- up to cosmetic details and a Fourier transform, \(C(\vec{l}_\perp)\) and \(\sigma_{\text{dipole}}\) are the same quantity
- there are fits of \(F_2\) at HERA based on the dipole model
Dilepton spectrum for pA at the LHC ($k_T^2 = 9 \text{ GeV}^2, y = 2.2$):

![Dilepton spectrum graph](image-url)

**Legend:**
- $\sigma_{d\sigma/dM_d/dp_T}$ (µb GeV$^{-3}$)
- $s^{1/2} = 8.8 \text{ TeV}$
- $A = 197$

- $MV_{mod}$ ($Q_s CGCfit m_q = 10 \text{ MeV}$)
- $MV_{mod}$ ($Q_s GBW$)
- $MV_{mod}$ asymp. ($Q_s CGCfit m_q = 10 \text{ MeV}$)
- $MV$ ($Q_s^2 = 8 \text{ GeV}^2$)
Production of lepton pairs

- $R_{pA}$ ratio in the MV model ($k^2 = 9 \text{ GeV}^2$):

![Graph showing $R_{pA}$ ratio in the MV model for different energies and collider types (RHIC and LHC, y=2.2 and y=3.2).]
Production of lepton pairs

- $R_{pA}$ ratio in the non-local gaussian model ($k^2 = 9 \text{ GeV}^2$):
Production of real photons

For dileptons with a large enough invariant mass, one does not need to worry about non-perturbative contributions in the fragmentation function $D_{q\rightarrow \gamma^*}$, because there is no collinear divergence when one integrates over the quark phase-space.

This is not so in the case of real photons:

$$
\frac{d\sigma_\gamma}{dz \, d^2 \vec{k}_\perp} = \frac{\pi R^2 \alpha_{em}}{8 \pi^4 k_\perp^2} \frac{1 + (1 - z)^2}{z} \int d^2 \vec{q}_\perp C(\vec{q}_\perp + \vec{k}_\perp) \frac{(\vec{q}_\perp + \vec{k}_\perp)^2}{[\vec{q}_\perp - \frac{1-z}{z} \vec{k}_\perp]^2}
$$

with $z$ the longitudinal momentum fraction of the photon relative to the incoming quark ($k^- = z p^-$)

- $z \vec{q}_\perp = (1 - z) \vec{k}_\perp$ if the photon and the final quark are collinear
- this divergence can be absorbed in the $D_{q\rightarrow \gamma}$ function:

$$
\frac{d\sigma_\gamma}{dz \, d^2 \vec{k}_\perp} = \frac{\pi R^2}{(2\pi)^2} C\left(\frac{k_\perp}{z}\right) D_{q\rightarrow \gamma}(z, k_\perp^2) \frac{d\sigma_q}{d^2 \vec{k}_\perp} \bigg|_{\frac{k_\perp}{z} = \frac{k_\perp^2}{z^2}}
$$

- this factorization is probably valid to all orders for pA collisions?
Photons in AA collisions

- Hadronic probes in AA collisions are not very good for testing the CGC because they undergo important final state effects. In fact, if thermalization occurs, the system largely forgets all the details about its initial conditions (at least for $p_\perp \lesssim 2$-3 GeV, which is the relevant region for saturation)

- The observation of photons produced at such early stages may bring some clue about the anisotropy/degree of thermalization of the early distribution of quarks, because real photons like to be produced collinearly to the emitter

- For production at mid-rapidity, and moderate $p_\perp$, both projectiles are probed at very small $x$

- For AA collisions, this means that the gluon densities of both projectiles are very large, and that multiple gluons from each nucleus can contribute

- Photons are produced from quarks... but the system is initially very poor in quarks/antiquarks. One must first solve the problem of quark production in a high gluon density environment
Calculation of observables

Required steps:
- **Solve the classical Yang-Mills equations** for arbitrary sources $\rho_{1,2}$. For the collision of two nuclei at high energy, the current in the YM equations reads

$$J^\mu = \delta^{\mu+}(x^-)\rho_1(\vec{x}_\perp) + \delta^{\mu-}(x^+)\rho_2(\vec{x}_\perp)$$

- so far, analytical solutions are known only if the source of one of the projectiles is treated at lowest order
- the full solution (all orders in the two sources) has been determined numerically

- **Calculate the matrix element** $M$ with the previously obtained gauge field in the background

  Note: the background field is now time-dependent, and transitions from the vacuum to populated states are non-zero

- **Perform the average over the sources** of each projectile, weighted by $W_{x_1}[\rho_1]$ and $W_{x_2}[\rho_2]$ respectively
Baltz, FG, McLerran, Peshier (2001)

- The fact that the background field is time-dependent (more exactly, has timelike modes) leads to several complications:
  - $|\text{in}\rangle$ and $|\text{out}\rangle$ states differ by more than a phase
  - The sum of the vacuum-to-vacuum diagrams is not a pure phase. It is important to have them in order to preserve unitarity
- Average multiplicities are somewhat simpler to calculate:

$$\omega_p \frac{dN}{d^3\vec{p}} \propto \Pi_{-+}(p, p) \propto \sum_{\text{states}} \int \sum_{f}$$

- no vacuum-to-vacuum diagrams for retarded amplitudes
- for gluon production at the classical level, this is just the square of the gauge field in Fourier space (amputated of its last leg)
Gluon production

- At the classical level, the gluon spectrum is given directly by the retarded solution of Yang-Mills equations:

\[ \omega_P \frac{dN_g}{d^3 \vec{p}} \bigg|_{\text{classical}} \propto \frac{1}{16\pi^3} \sum_{\lambda} \left| p^2 \epsilon^{(\lambda)}_{\mu}(\vec{p}) A^\mu(p) \right|^2 \]


- The spectrum obtained by this method reproduces the pQCD tail in \( p_\perp^{-4} \), and leads to a regular spectrum at small \( p_\perp \)

- Overall good agreement with RHIC measurements of the multiplicity, with a reasonable value of \( Q^2_s \sim 2 \text{ GeV}^2 \)
Quark production


- The inclusive quark spectrum can be obtained from the retarded propagator of the quark in the classical color field:

\[ \omega_p \frac{dN_Q}{d^3\vec{p}} \propto \int_{\vec{q}} \frac{d^3\vec{q}}{(2\pi)^3} |\bar{u}(\vec{p})T_{ret}(p, -q)v(q)|^2 \]

- The calculation can be carried out analytically if one of the two sources is weak [breaking of \( k_\perp \)-factorization]

- Since the gauge field is known numerically to all orders in both sources, it is possible to reformulate the problem of quark production in a way which is suitable for a numerical approach
Quark production

Alternate representation of the amplitude:

\[
\bar{u}(\vec{p}) T_{\text{ret}}(p, -q) v(q) = \lim_{t \to +\infty} \int d^3 \vec{x} \, e^{i p \cdot x} u^\dagger(\vec{p}) \psi_q(t, \vec{x})
\]

\[
(i \partial_x - g A(x) - m) \psi_q(x) = 0, \quad \psi_q(t, \vec{x}) \to v(q) e^{i q \cdot x}
\]

The problem amounts to solving numerically the Dirac equation in a known (numerically) color background field, and to project out the solution on a free spinor.

It is a much harder problem than gluon production: even in a boost invariant background field, there is still a non-trivial correlation between the rapidities of the quark and the antiquark.

CPU estimate: a few days on a TFlops-scale computer.
Photon production

- The inclusive photon spectrum is given in terms of the "forward" photon self-energy in the background gauge field:

\[
\omega_p \frac{dN_\gamma}{d^3 \vec{p}} = \frac{1}{16\pi^3} \sum_{\text{polarizations}} \epsilon_\mu(\vec{p}) \epsilon_\nu(\vec{p}) \Pi^{\mu\nu}_{+-}(p,p)
\]

- Diagrams:

- At leading log accuracy, the photon can only be emitted at the end of an open quark line. ▶ it is sufficient to convolute the inclusive quark spectrum with a vacuum fragmentation function.
Photon production

- Beyond leading log, one needs the components $D_{-+}$, $D_{--}$ and $D_{++}$ of the quark propagator in the presence of a strong color field which is known only numerically.

- **Limitation:** when solving numerically the Dirac equation, only retarded boundary conditions can be treated “easily”

- For propagators in a background field at tree level:

  \[
  D_{-+} = D_R \ast D_{R}^{0-1} \ast D_{-+} \ast D_{A}^{0-1} \ast D_A \\
  D_{++} = \frac{1}{2} \left[ D_R \ast D_{R}^{0-1} \ast D_{s}^{0} \ast D_{A}^{0-1} \ast D_A + (D_R + D_A) \right] \\
  D_{--} = \frac{1}{2} \left[ D_R \ast D_{R}^{0-1} \ast D_{s}^{0} \ast D_{A}^{0-1} \ast D_A - (D_R + D_A) \right]
  \]

  with: $D_s^{0} = D_{++}^{0} + D_{-+}^{0}$

- Since the $D_{++}$ and $D_{--}$ propagators are only needed with the same initial and final points, one can drop the $D_R + D_A$ piece they contain.
Photon production

In the problem of quark production, one solves numerically the Dirac equation with retarded boundary conditions. The previous propagators can be written in terms of these solutions:

\[
D_{+ -} (x, y) = \int \frac{d^3 \vec{k}}{(2\pi)^3 2\omega_k} \sum_{\text{spin}} \overline{\psi}^{(-)} (y) \psi^{(-)} (x)
\]

\[
D_{- +} (x, y) = \int \frac{d^3 \vec{k}}{(2\pi)^3 2\omega_k} \sum_{\text{spin}} \overline{\psi}^{(+)} (y) \psi^{(+)} (x)
\]

\[
D_{\pm \pm} (x, x) = \frac{1}{2} \int \frac{d^3 \vec{k}}{(2\pi)^3 2\omega_k} \sum_{\text{spin}} \left[ \overline{\psi}^{(+)} (x) \psi^{(+)} (x) + \overline{\psi}^{(-)} (x) \psi^{(-)} (x) \right]
\]

Now everything is expressed in terms of objects that are calculable numerically, but the size of the task is daunting...
Conclusions

- The LHC will be a very good playground for testing small-$x$ physics, and to test the predictions of the CGC.

- In pA collisions, in the direction of the fragmentation region of the proton, one probes the proton at moderate $x$ and the nucleus at very small $x$.
  - One can use a hybrid description: the proton in terms of ordinary PDFs, and the nucleus by means of the Color Glass Condensate.
  - Photons or dileptons are an alternative to hadronic probes for testing the physics of saturation/shadowing.

- In AA collisions, hadronic probes are not very well suited for testing this physics because of the final state effects.
  - Photons offer a possibility to achieve this, because they do not rescatter. In particular, they could potentially help to assess the anisotropy of the initial distribution of quarks.
  - Unfortunately, the calculation is much more difficult than that of gluon production...