FINAL EXAM

PHYS 625 (Spring 2011), 05/09/11

Name:

Signature:

Duration: 120 minutes
Show all your work for full/partial credit
Quote your answers in units of MeV *(or* GeV*) and* fm*, or combinations thereof*

No.	Points
1	
2	
3	
4	
5	
Sum	

1.) *Central NN Potential from Meson Exchange* *(8+8+4+4 pts.)*

In momentum space, the meson-exchange model for the nucleon-nucleon interaction is given by meson propagators and coupling constants as

$$V_\alpha(q) = \pm g_\alpha^2 \, \frac{1}{\vec{q}^{\,2} + m_\alpha^2} \ , \tag{1}$$

where spin-isospin factors have been suppressed.

(a) Calculate the meson exchange potential in coordinate space using a Fourier transform.

(b) In the following, consider the contributions from (attractive) sigma and (repulsive) omega exchange with g_σ=10.5, m_σ=550 MeV and g_ω=14.2, m_ω=782 MeV. Calculate the values of $V_\alpha(r)$ for both contributions, as well as for the total, at distances r=0.5, 1, and 1.5 fm.

(c) Sketch in a graph $V(r) = V_\sigma(r) + V_\omega(r)$.

(d) The many body-theory using non-relativistic nucleon-nucleon potentials is referred to as Brueckner-Bethe-Goldstone (BBG) theory. Explain at least one success *and* one drawback of this approach.

2.) *Relativistic Mean-Field Model for Nuclear Matter* *(10+8+4 pts.)*

In mean-field theory, the energy density of nuclear matter in the relativistic σ-ω model at zero temperature has been found to be

$$\frac{E}{V} = \epsilon(\varrho_N;\phi_0) = \frac{1}{2}m_\sigma^2\phi_0^2 + \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\varrho_N^2 + d_{\rm SI}\int_0^{k_F}\frac{d^3k}{(2\pi)^3}E_k^* \tag{2}$$

with nucleon density $\varrho_N{=}2k_F^3/(3\pi^2)$, spin-isospin degeneracy $d_{\rm SI}{=}4$, in-medium nucleon mass $m_N^*{=}m_N{-}g_\sigma\phi_0$ and in-medium nucleon energy $E_k^*{=}[(m_N^*)^2+\vec{k}^2]^{1/2}$.

(a) Using $\partial\epsilon/\partial\varrho_N = (P+\epsilon)/\varrho_N$ show that the pressure can be written as

$$P(\varrho_N;\phi_0) = -\frac{1}{2}m_\sigma^2\phi_0^2 + \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\varrho_N^2 + d_{\rm SI}\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\frac{\vec{k}^2}{3E_k^*}\ . \tag{3}$$

Also use $d\varrho_N = (2/\pi^2)k_F^2dk_F$ and a partial integration.

(b) Using a minimization condition of the energy, $E=\epsilon V$, at fixed ϱ_N (i.e., fixed A and V), show that the scalar mean field is given by

$$\phi_0 = \frac{g_\sigma}{m_\sigma^2}\varrho_s \quad \text{with} \quad \varrho_s = d_{\rm SI}\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\frac{m_N^*}{E_k^*}. \tag{4}$$

(c) Rewrite the pressure in terms of ϱ_s and argue why nuclear saturation in the RMF σ-ω model is a relativistic effect.

3.) *Constituent Quark Model* *(10+6+4 pts.)*

In the constituent-quark model the spin-flavor wave function of a proton with spin up is given by

$$|p^{\uparrow}\rangle = \frac{1}{\sqrt{18}}[uud(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) + udu(\uparrow\uparrow\downarrow + \downarrow\uparrow\uparrow - 2\uparrow\downarrow\uparrow) + duu(\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow - 2\downarrow\uparrow\uparrow)] \ , \quad (5)$$

with notation $uud(\uparrow\downarrow\uparrow) \equiv u^{\uparrow}u^{\downarrow}d^{\uparrow}$, and so on.

(a) Calculate the magnetic moment, $\mu_p = \langle p^{\uparrow}|\hat{\mu}_h|p^{\uparrow}\rangle$, of the proton using the hadronic operator

$$\hat{\mu}_h = \sum_i \mu_i \ \sigma_z^i \quad , \quad \mu_i = \frac{e_i}{2m_i} \ . \quad (6)$$

The sum is over the quark constituents in the hadron and $\sigma_z^i = diag(1, -1)$ is the quark-spin Pauli matrix with eigenvalues ± 1 for spin-up/-down. Calculate and express your result in terms of the up- and down-quark magnetic moments, $\mu_{u,d} = \frac{e_{u,d}}{2m_{u,d}}$ (*not* in units of 1/GeV).

(b) Isospin symmetry implies the neutron wave function to result from the one of the proton via the exchange $u \leftrightarrow d$ and $m_u = m_d$. Use your result from (a) to immediately obtain the neutron magnetic moment, μ_n, in terms of μ_u and μ_d. Then use e_u=2/3 and e_d=−1/3 to calculate the ratio μ_n/μ_p and compare it to the experimental value of −0.685.

(c) What is the overall symmetry of the proton wave function under exchange of any two quarks? Is that a problem, and if so, how is it resolved?

4.) *QCD Vacuum* *(6+10 pts.)*

When treating the effective quark Lagrangian of the Nambu-Jona-Lasinio (NJL) model in mean-field approximation one obtains the (non-regularized) free energy as

$$\Omega(\chi_0) = \frac{(m_q^* - m_q)^2}{4G} - d_{SCF} \int_0^\Lambda \frac{4\pi p^2 dp}{(2\pi)^3} \omega_p^* \ , \tag{7}$$

where $m_q^* = m_q - 2G\chi_0$ and m_q are the effective and bare quark mass, respectively, χ_0 is the quark-antiquark condensate, d_{SCF} the spin-color-flavor degeneracy of the light quarks, Λ a sharp 3-momentum cutoff and $\omega_p^* = (p^2 + m_q^*)^{1/2}$ the quark energy. Assume 2 light flavors with m_q=0.

(a) Calculate the effective quark mass (in MeV) for a coupling constant $G = 0.3\,\mathrm{fm}^2$ and a quark condensate of $\chi_0 = -4\ \mathrm{fm}^{-3}$.

(b) Calculate the (regularized) nonperturbative vacuum pressure, $P = -\bar{\Omega}$ (in GeV/fm^{-3}), assuming Λ=0.6 GeV. Make sure to subtract the contribution for vanishing effective quark mass, Ω_0, i.e., in the absence of any condensate.

5.) *QCD Phase Diagram* *(18 pts.)*

Assume the pressure and energy density of the nonperturbative QCD vacuum to be given by $P = -\epsilon = 0.3\,\mathrm{GeV/fm^3}$.

(a) Calculate the minimum temperature for a non-interacting massless 2-flavor quark-gluon plasma (QGP) at vanishing chemical potential ($\mu_q = 0$, i.e., equal number of quarks and antiquarks) to overcome the vacuum pressure. What is the energy density (in $\mathrm{GeV/fm^3}$) of the QGP at this temperature? Make sure to include the appropriate spin-color-flavor-antiparticle degeneracy for quarks and gluons. (Use the Boltzmann approximation in this part).

(b) Repeat part (a) for a non-intertacting quark gas at zero temperature to calculate the minimum chemical potential and pertinent energy density.

Formula Sheet

Fourier Transform and Residue Theorem:

$$f(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} \exp(-i\vec{q}\cdot\vec{r}) f(\vec{q}) \quad , \quad \int\limits_{\text{contour}} \frac{f(q)}{q - iq_0} = 2\pi i f(iq_0) \quad \text{with } f(q) \text{ regular}$$

Partial Integration:

$$\int uv' = uv - \int u'v$$

Pressure and energy density of ideal gas:

$$P = deg \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^{\,2}}{3E_k} f(E_k;\mu,T) \quad , \quad \epsilon = deg \int \frac{d^3k}{(2\pi)^3} E_k f(E_k;\mu,T)$$

with $f = 1/(\exp[(E_k - \mu)/T] \pm 1)$ Fermi/Bose, or $f = \exp[-(E_k - \mu)/T]$ Boltzmann

Indefinite integral with $X = x^2 + a^2$:

$$\int dx\, x^2 \sqrt{X} = \frac{x}{4} X^{3/2} - \frac{a^2}{8}[x\sqrt{X} + a^2 \ln(x + \sqrt{X})] + \text{const}$$

Conversion factor: $\hbar c = 197$ MeV fm $= 0.197$ GeV fm