FINAL EXAM

PHYS 625 (Fall 2013), 12/10/13

Name:

Signature:

Duration: 120 minutes
Show all your work for full/partial credit
Quote your answers in units of MeV *(or* GeV*) and* fm*, or combinations thereof*

No.	Points
1	
2	
3	
4	
5	
6	
Sum	

1.) *Conceptual Questions (no calculations required)* *(18 pts.)*

(A) Sketch the central coordinate-space potential between two nucleons, $V_{NN}^{\text{cent}}(r)$.
Give a microscopic explanation of its long- and short-distance components.
Explain qualitatively how it generates nuclear saturation and renders the volume of nuclei proportional to the atomic number, A.

(B) What problem arises when calculating the expectation value of the two-body NN potential in nuclear matter, $\langle \Psi_A | V_{NN}(r) | \Psi_A \rangle$, using Fermis gas wave functions, Ψ_A?
How is that problem solved in Brueckner-Bethe-Goldstone theory?

(C) Briefly explain the mean-field approximation in the relativistic field-theoretic approach to nuclear matter, and how relativistic effects lead to nuclear saturation in the σ-ω model.

2.) *Empirical Nuclear Matter* *(18 pts.)*

Infinite nuclear matter at saturation is characterized by a Fermi momentum of $k_F = 265\,\text{MeV}$ and a binding energy per nucleon of $E_B/A = -16\,\text{MeV}$.

(a) Calculate the saturation density ϱ_0 [fm^{-3}] by integrating up the nucleon momentum phase space. Account for the spin-isospin degeneracy of nucleons. What is the average distance, d_{NN}, between two nucleons?

(b) Calculate the average kinetic energy, $\langle \frac{k^2}{2M_N} \rangle$, of nucleons at ϱ_0, and infer the average potential energy of each nucleon to recover the empirical binding energy.

(c) If each nucleon has six nearest neighbors contributing to its average potential energy, what should be the value of the two-nucleon potential, $V_{NN}(r)$, at a separation $r = d_{NN}$?

3.) *Liquid Drop Model (LDM) of Nuclei* *(5+4+4+2+2 pts.)*

The empirical Weizsäcker formula for the binding energy of nuclei is given by

$$E_B = \sum_{i=1}^{5} E_i = -a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A} + a_5 \frac{\lambda}{A^{3/4}} \tag{1}$$

with A: nuclear mass number, Z: nuclear charge (in units of e), a_1=15.75 MeV, a_2=17.8 MeV, a_3=0.71 MeV, a_4=23.7 MeV and a_5=34 MeV with λ=-1,0,1 for e-e,e-o,o-o nuclei.

(a) Briefly discuss the physical motivation (A and Z dependence) for each term.

(b) Derive the value Z^* for the charge which minimizes the binding energy for fixed A. Sketch the resulting valley of stability, $Z^*(N)$.

(c) Sketch $|E_B(A)|/A$ by subsequently adding the terms of the LDM in numerical order for i=1-4. In each step indicate whether $|E_B(A)|/A$ develops a maximum.

(d) When fissioning ${}^{235}_{92}U \rightarrow {}^{137}_{55}Cs + {}^{96}_{37}Rb + N_1 n + N_2 p$, how many free neutrons (N_1) and protons (N_2) are emitted, and where does the released binding energy go?

(e) Which important feature of nuclear structure is not explained by the LDM?

4.) *Isospin Invariance of πN Lagrangian* *(14 pts.)*

A simple π-N-N interaction Lagrangian is given by

$$\mathcal{L}_{\pi NN} = g_{\pi NN}\; \bar{\psi}_N\, i\gamma_5\, \vec{\pi}\cdot\vec{\tau}\,\psi_N \;, \tag{2}$$

where arrows indicate vectors in isospin space (the nucleon spinors are doublets in this space).

(a) Show that the above Lagrangian is invariant under rotations in isospin space by applying an infinitesimal rotation to all field operators about an angle $\alpha \ll 1$,
$\psi_N \to (1 - i\vec{\alpha}\cdot\vec{\tau}/2)\psi_N \;, \quad \pi \to (1 + \vec{\alpha}\,\times)\;\vec{\pi} \;\;,$
and verifying the invariance to leading order in α.

(b) Show that the above Lagrangian predicts relations between the physical couplings as

$$g_{pp\pi^0} = -g_{nn\pi^0} = \frac{1}{\sqrt{2}} g_{pn\pi^+} = \frac{1}{\sqrt{2}} g_{pn\pi^-} \tag{3}$$

where the physical (charged and neutral) pion fields are related to the cartesian ones, $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$, as $\pi^\pm = (\pi_1 \pm i\pi_2)/\sqrt{2}$ and $\pi^0 = \pi_3$.

5.) *Running Coupling in Quantum Chromodynamics (QCD)* *(15 pts.)*

The dependence of the QCD coupling parameter on momentum transfer, Q^2, can be written as

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 - \frac{\alpha_s(\mu^2)}{12\pi}(2N_f - 11N_c)\ \ln\left(\frac{Q^2}{\mu^2}\right)}\ , \tag{4}$$

where its value at a scale μ^2 needs to be determined experimentally, and N_f: number of quark flavors, N_c=3: number of colors.

(a) Explain why the running of α_s with distance $r \sim 1/Q$ is counter-intuitive. What is the origin of this effect? (*hint*: gluon self-coupling)

(b) Show that Eq. (4) can be rewritten as

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)\ \ln\left(\frac{Q^2}{\Lambda^2_{\mathrm{QCD}}}\right)}\ , \tag{5}$$

where Λ^2_{QCD} is formally defined as the scale where $\alpha_s(Q^2)$ diverges.

(c) For $\Lambda_{\mathrm{QCD}} = 0.2\,\mathrm{GeV}$, how large is $\alpha_s(Q^2)$ at Q=1,10,100 GeV? Sketch $\alpha_s(Q^2)$ in a graph.

6.) *Nambu Jona-Lasinio (NJL) Model for Vacuum Structure* *(18 pts.)*

In mean-field approximation, the free energy density of the NJL model at finite temperature, T, and quark chemical potential, μ_q, is given by

$$\frac{\Omega(\mu_q, T; \chi_0)}{V} = G\chi_0^2 - d_{scf} \int \frac{d^3p}{(2\pi)^3} \omega_p^* - T d_{scf} \int \frac{d^3p}{(2\pi)^3} [\ln(1 + \mathrm{e}^{-(\omega_p^* - \mu_q)/T}) + \ln(1 + \mathrm{e}^{-(\omega_p^* + \mu_q)/T})] \tag{6}$$

with $d_{scf} = N_s N_c N_f$: quark degeneracy, $\omega_p^* = (p^2 + m_q^{*2})^{1/2}$: quark energy, G: 4-quark coupling, $\chi_0 = \langle 0|\bar{q}q|0\rangle$: quark-antiquark condensate, $m_q^* = m_q - 2G\chi_0$: constituent quark mass.

(a) Take the $T \to 0$ limit of Eq. (6) and use the quark density in writing your final result. (*hint*: start by evaluating $T\ln(1 + \mathrm{e}^{-x/T})$ for $T \to 0$ for positive/negative x)

(b) Take the limit of vanishing chemical potential of your result from part (a) to show that

$$\frac{\Omega(m_q^*)}{V} = \frac{(m_q^* - m_q)^2}{4G} - d_{scf} \int \frac{d^3p}{(2\pi)^3} \omega_p^* \ . \tag{7}$$

Briefly interpret the physical origin of the two terms.

(c) Derive the self-consistency gap equation for the constituent quark mass. What additional ingredient is necessary to render the equation meaningful, and what is the physical interpretation of this ingredient?

Formula Sheet

Commutation relations

$$[\tau_i, \tau_j] = 2i\varepsilon_{ijk}\tau_k$$

Hadron masses:

$$m_N = 940\,\mathrm{MeV},\ m_\pi = 140\,\mathrm{MeV},\ m_\sigma = 550\,\mathrm{MeV}\ m_\omega = 782\,\mathrm{MeV}$$

Conversion factor: $\hbar c = 197\ \mathrm{MeV\,fm} = 0.197\ \mathrm{GeV\,fm}$