Midterm Exam-3 Spring '16

1.) *Multiple Choice* *(18 pts.)*
For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

(a) The torque on an object depends on the angle under which the force is applied.
TRUE FALSE

(b) When a solid cylinder is rolling without slipping, 50% of its total kinetic energy is in the rotational component of the motion.
TRUE FALSE

(c) Angular momentum is a vector quantity.
TRUE FALSE

(d) The center of gravity of an object can lie outside the material of the object.
TRUE FALSE

(e) When you increase the amplitude of a given harmonic oscillator, its oscillation frequency will not change.
TRUE FALSE

(f) When a simple harmonic oscillator reaches half of its amplitude, its kinetic energy and potential energy are equal.
TRUE FALSE

No.	Points
1	TW
2	JW
3	YZ
4	RR
5	CH
Sum	

3.) *Angular Momentum Conservation* *(20 pts.)*

A man is sitting on a rotating stool spinning with an initial angular speed of $3\,rad/s$. The stool plus man have a moment of inertia of $4\,kg\,m^2$, assumed to be constant. In addition, the man is holding a weight of mass $2.5\,kg$ (assumed to be a point mass) in each of his hands, initially at a distance of $1.2\,m$ away from the axis of rotation. He then pulls the weights toward his body, to a distance of $0.3\,m$ from the axis of rotation.

(a) Calculate the angular velocity of the system after the weights have been pulled in.

(b) Calculate the rotational kinetic energy before and after pulling in the weights. Comment on the difference, if any.

(a) $L_0 = L_1$

$I_0 w_0 = I_1 w_1$

$I_0 = I_{man} + 2 m_w r_0^2$

$I_1 = I_{man} + 2 m_w r_1^2$

$$\boxed{w_1 = w_0 \frac{I_0}{I_1} = 3 \frac{(4+5\cdot 1.2^2)}{(4+5\cdot 0.3^2)} = 7.55\ rad/s}$$

(b) $\boxed{K_0 = \frac{1}{2} I_0 w_0^2 = 50.4\ J}$

$\boxed{K_1 = \frac{1}{2} I_1 w_1^2 = 127\ J}$

larger because of "internal" work

2.) *Mechanical Energy in Rotational Motion* (10+8 pts.)

A basketball (30 cm in diameter, approximated by a hollow thin-walled sphere) is rolling up a hill without slipping and without any friction losses. It comes to a stop at a maximum height of 8 m above the starting point.

(a) What was the angular speed of the ball at the bottom?

(b) What is the percentage partition of translational and rotational kinetic energy?

(a) $E_o = E_f$

$\frac{1}{2} m v_o^2 + \frac{1}{2} I \omega_o^2 = mgh$

$I = \frac{2}{3} m r^2$

$v_o = \omega_o r$

$\frac{1}{2} m (\omega_o r)^2 + \frac{1}{2} \frac{2}{3} m r^2 \omega_o^2 = mgh$

$\left(\frac{1}{2} + \frac{1}{3}\right) \omega_o^2 r^2 = gh$

$\boxed{\omega_o = \sqrt{6gh/5r^2}} = \boxed{64.7 \frac{rad}{s}}$

(b) translational $\frac{\frac{1}{2}}{5/6} = \frac{3}{5} = 60\%$

rotational $\frac{1/3}{5/6} = \frac{2}{5} = 40\%$

4.) *Force and Torque Equilibrium* (24 pts.)

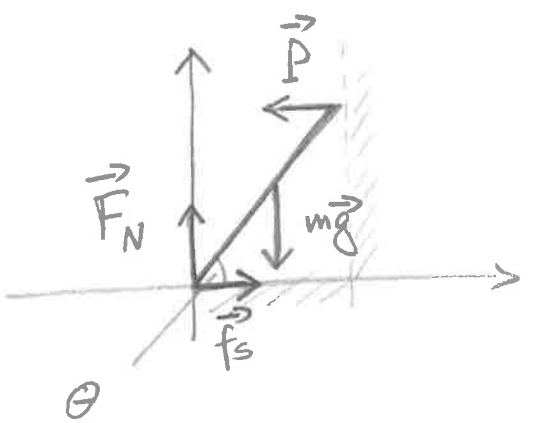
A ladder is leaning against a wall as shown in the sketch below. Account for the following forces acting on the ladder: weight force, normal force from the ground, normal force from the wall, and a static friction force at the bottom of the ladder with a static friction coefficient of μ_s=0.5.

(a) Draw a free-body diagram of the ladder.

(b) Write down the equations for force equilibrium as well as torque equilibrium choosing the bottom of the ladder as the (would-be) axis of rotation.

(c) Calculate the minimal angle θ before the ladder starts slipping.

(a)



(b) $\sum F_x = f_s - P = 0 \Rightarrow f_s = P$

$\sum F_y = F_N - mg = 0 \Rightarrow F_N = mg$

$\sum \tau = P\,\ell_P - \ell_g\, mg = 0\,,\quad \ell_P = \ell \sin\theta,\quad \ell_g = \frac{\ell}{2}\cos\theta$

(c) $0 = P\,\ell \sin\theta - \frac{\ell}{2} mg \cos\theta \qquad P = f_s = \mu_s F_N = \mu_s mg$

$= \mu_s mg\,\ell \sin\theta - \frac{\ell}{2} mg \cos\theta$

$\Rightarrow 0 = \mu_s \sin\theta - \frac{1}{2}\cos\theta$

$\frac{1}{2}\cos\theta = \mu_s \sin\theta \Rightarrow \frac{1}{2\mu_s} = \tan\theta$

$\boxed{\theta = \tan^{-1}\left(\frac{1}{2\mu_s}\right) = 45^\circ}$

5.) *Simple Harmonic Motion* *(20 pts.)*

A simple harmonic oscillator consists of a mass of 4 kg and an ideal spring of unknown spring constant. The oscillation has a period of 0.8 s and a maximum speed of 1.2 m/s. Calculate

(a) the spring constant.

(b) the amplitude of the oscillation.

(a) $$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

$$\frac{k}{m} = \frac{4\pi^2}{T^2} \Rightarrow \boxed{k = \frac{4\pi^2 m}{T^2} = 247\ \frac{N}{m}}$$

(b) $$V_{max} = A\omega$$

$$\boxed{A = \frac{V_{max}}{\omega} = \frac{V_{max}\,T}{2\pi} = 0.153\ m}$$