Ex-3 Sol. S15

1.) *Multiple Choice* (18 pts.)

For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

(a) The angular acceleration of a rotating rigid body is different for different pieces of that body.
TRUE FALSE

(b) If a solid cylinder is rolling without slipping at constant speed, its rotational kinetic energy is smaller than its translational kinetic energy.
TRUE FALSE

(c) When a solid cylinder and a hoop roll down the same hill, both staring from rest, the cylinder will reach the bottom first.
TRUE FALSE

(d) Angular momentum is a scalar quantity.
TRUE FALSE

(e) The center of mass of an object cannot be located outside the actual object.
TRUE FALSE

(f) The dynamical origin of simple harmonic motion is Hooke's law for an elastic restoring force.
TRUE FALSE

No.	Points
1	CH
2	ZW
3	SC
4	LS
5.	RR
Sum	

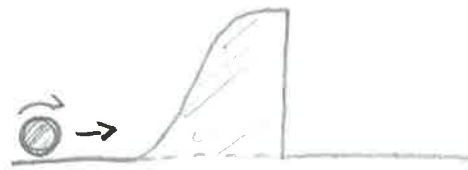
2.) *Kinetic Energy* **(12+10** *pts.)*

A tennis ball (hollow thin-walled sphere of 5 cm) is rolling up a hill without slipping. The ball's initial speed at the bottom of the hill is 11 m/s, (12) and the height of the hill is 10 m. At the top of the hill the ball launches off a cliff horizontally. The ground level on both sides of the hill/cliff is identical. Neglect friction and air drag losses.

(a) What are the ball's angular and translational speeds at the top of the hill?

(b) What is the speed of the ball just before it lands at the bottom of the cliff? In view of your result, comment on whether mechanical energy is conserved throughout the entire motion.

(a) $E_0 = E_1$ $\qquad I = \frac{2}{3}mr^2, \quad \omega = \frac{v}{r}$



$$\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 + mgh_1$$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\frac{v_0^2}{r^2} = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\frac{v_1^2}{r^2} + mgh_1$$

$$\frac{1}{2}v_0^2 + \frac{1}{3}v_0^2 = \frac{1}{2}v_1^2 + \frac{1}{3}v_1^2 + gh_1$$

$$5/6\, v_0^2 = \frac{5}{6}v_1^2 + gh_1 \quad \Rightarrow \quad \boxed{v_1 = \sqrt{v_0^2 - \frac{6}{5}gh_1} = 1.84\,\frac{m}{s}} \; (5.14)$$

$$\boxed{\omega_1 = \frac{v_1}{r} = 36.9\,\frac{rad}{s}} \; (102.8)$$

(b) $E_1 = E_2$ $\qquad \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + \cancel{\frac{1}{2}I\omega_1^2}$!

$$\boxed{v_2 = \sqrt{v_1^2 + 2gh_1} = 14.1\,\frac{m}{s}} \; (14.9)$$

Most of the original rotational energy is now in translational kinetic energy.

3.) *Rotational Dynamics and Power* (20 pts.)

A merry-go-round (solid disk of mass 90 kg and radius 2 m) is originally at rest (but free to rotate in the horizontal plane about its center). A child applies an external force of 20 N tangentially to the outer edge of the merry-go-round, for a duration of 15 s. (30N)

(a) What is the final angular velocity of the merry-go-round?

(b) What is the average power supplied by the child?

(a) $\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{Fr}{\frac{1}{2}mr^2} = \frac{2F}{mr} = 0.22 \frac{rad}{s^2}$ (0.33)

$\boxed{\omega = \omega_0 + \alpha t = 3.33 \frac{rad}{s}}$ (5.0)

(b) $\boxed{P_R = \frac{W_R}{\Delta t} = \frac{\frac{1}{2}I\omega^2}{\Delta t} = 66.7\ W}$ (150)

4.) *Equilibrium* (8+12 pts.)

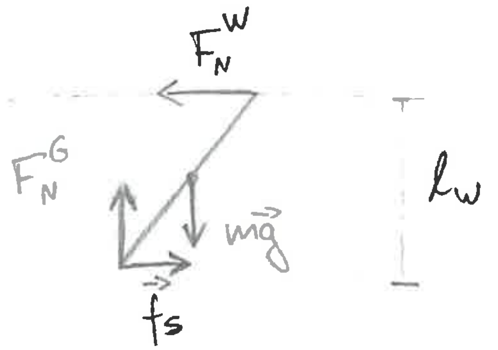
A uniform ladder is leaning against a smooth (frcitionless) vertical wall at an inclination angle of (63°) 70° above the horizontal. There is friction between the bottom of the ladder and the horizontal ground.

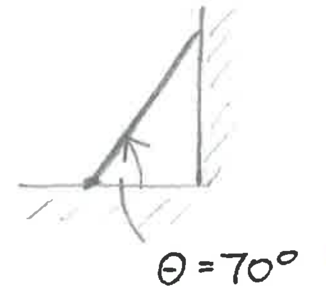
$(m = 17 kg)$

(a) Draw the free body diagram of the ladder.

(b) Apply force and torque equilibrium to find the static friction force between the ladder and the ground.

(a)





(b)

$$\sum F_x = 0 = f_s - F_N^W$$

$$\sum F_y = 0 = F_N^G - mg$$

$$\sum \tau = 0 = -mg\, l_g + F_N^W l_w$$

$$= -mg \frac{l_w}{2\tan\theta} + f_s l_w$$

θ $\frac{1}{2} l_w$

$$l_g = \frac{\frac{1}{2} l_w}{\tan\theta}$$

$$\Rightarrow \boxed{f_s = \frac{mg}{2\tan\theta} = 30.3\,N} \quad (48.1)$$

5.) *Simple Harmonic Motion* (20 pts.)

A block of mass $0.5\,kg$ is attached to an ideal spring and performs simple harmonic motion with a period of $1.1\,s$ and an amplitude of $0.15\,m$.

(0.8)

(a) Find the spring constant.

(b) Find the maximal speed and acceleration of the block.

(a) $$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{T^2}{4\pi^2} = \frac{m}{k} \Rightarrow \boxed{k = \frac{4\pi^2 m}{T^2} = 16.3\,\frac{N}{m}} \quad (30.8)$$

(b) $$\boxed{V_{max} = A\omega = \frac{2\pi A}{T} = 0.86\,m/s} \quad (1.18)$$

$$\boxed{a_{max} = \frac{k\,x_{max}}{m} = 4.9\,\frac{m}{s^2}} \quad (9.25)$$