Exam-3 Solution Key

1.) *Multiple Choice* *(18 pts.)*

For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

(a) If a solid disk is rotating at constant angular velocity, different points on the disk generally have different tangential speeds.
TRUE FALSE

(b) If a solid cylinder is rolling without slipping at constant speed, its rotational kinetic energy is larger than its translational kinetic energy.
TRUE FALSE

(c) When a solid cylinder and a hoop roll down the same hill, both staring from rest, the cylinder will reach the bottom first.
TRUE FALSE

(d) The moment of inertia of a rigid object depends on the axis about which the object is rotating.
TRUE FALSE

(e) Torque is a scalar quantity.
TRUE FALSE

(f) In simple harmonic motion, the total mechanical energy $[E = 0.5(kx^2 + mv^2)]$ is conserved even though x and v vary with time.
TRUE FALSE

No.	Points
1	IS
2	AL
3	BY
4	RR
5	YZ
Sum	

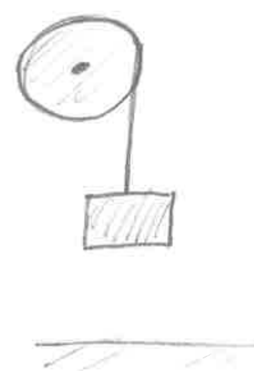
2.) *Rotational Energy and Dynamics* *(10+8+5 pts.)*

A massless string is wrapped around the outer rim of a solid disk (mass 70 kg, radius 30 cm) which is fixed in place but is free to rotate about its center, see sketch below. A package of mass 25 kg is attached to the end of the string and released from rest, 4.5m above the ground.

(a) What is the speed of the package just before it hits the ground?

(b) What is the acceleration of the package and the angular acceleration of the disk?

(c) What is the net torque on the disk?



(a) $E_i = E_f$

$mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$

$I = \frac{1}{2} M r^2 \qquad \omega = V/r$

$\Rightarrow mgh = \frac{1}{4} M v^2 + \frac{1}{2} m v^2 = \left(\frac{1}{4} M + \frac{1}{2} m\right) v^2$

$\Rightarrow \boxed{V = \sqrt{\frac{2m}{\frac{1}{2}M + m} gh}} = \boxed{6.06 \ m/s}$

(b) $V^2 = V_0^2 + 2ah \Rightarrow \boxed{a = \frac{V^2}{2h}} = \boxed{4.08 \ m/s^2}$

$\boxed{\alpha = \frac{a}{r}} = \boxed{13.6 \ rad/s^2}$

(c) $\boxed{\tau = I\alpha} = \left(\frac{1}{2} M r^2\right) \alpha = \boxed{42.9 \ Nm}$

3.) *Angular Momentum* (12+3pts.)

A small wooden turntable (mass 30 g) is rotating in the horizontal plane about its center at 2 revolutions per second. A honey bee (mass 0.8 g) lands vertically on the edge of the turntable (assume the bee's velocity just prior to landing to be zero).

(a) Calculate the angular speed of turntable plus bee after landing.

(b) Do you expect energy to be dissipated? ☒ yes ☐ no

(a) $L_f = L_i$

$$\left(\frac{1}{2}Mr^2 + mr^2\right)\omega_f = \frac{1}{2}Mr^2\,\omega_i \qquad \omega_i = 2\pi f_i$$

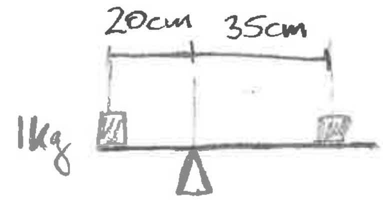
$$\Rightarrow \boxed{\omega_f = \frac{Mr^2}{(Mr^2 + 2mr^2)}\,\omega_i = \frac{M}{(M+2m)}\,2\pi f_i = 8.2 \text{ rad/s}}$$

4.) *Equilibrium* *(8+16 pts.)*

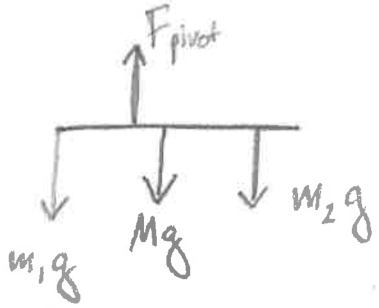
A scales (a uniform bar of length 60 cm and mass 0.5 kg) is loaded with a weight of 2 kg at 20 cm to the left of the pivot point, and an unkown weight at 35 cm to the right of the pivot point (see sketch below). The bar is in equilibrium.

(a) Draw the free body diagram of the bar.

(b) Find the the unkown mass.



(a)



(b) Relative to pivot point

$$0 = \sum_i \tau = F_1 \ell_1 - F_w \ell_w - F_2 \ell_2$$

$$= m_1 g \ell_1 - M g \ell_w - m_2 g \ell_2$$

$$\Rightarrow \quad m_2 = \frac{m_1 \ell_1 - M \ell_w}{\ell_2}$$

$\ell_1 = 0.2\,m, \quad \ell_2 = 0.35\,m, \quad \ell_w = 0.1\,m$

$$\boxed{m_2 = 1\,kg}$$

5.) *Simple Harmonic Motion* (*20 pts.*)

On a frictionless horizontal surface, a block (mass 2.5 kg) is attached to an ideal spring and performs simple harmonic motion with a frequency of 0.8 Hz.

(a) Find the spring constant.

(b) If the amplitude of the motion is 15 cm, what is the maximal speed reached by the block?

(a) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\Rightarrow \boxed{k = (2\pi f)^2 m} = \boxed{63.2 \frac{N}{m}}$

(b) $E_1 = E_2$

$\frac{1}{2} k A^2 = \frac{1}{2} m v_{max}^2 \quad \Rightarrow \quad \boxed{v_{max} = \sqrt{\frac{k}{m}}\, A = 2\pi f A = 0.75 \frac{m}{s}}$