

# EXAM-3

PHYS 201 (Spring 2008), 04/18/08

Name: Solution Key

Lab-Sect. no.:

Signature:

*Duration: 50 minutes*

*Show all your work for full/partial credit!*

*Include the correct units in your final answers for full credit!*

*Unless otherwise stated, quote your results in SI units!*

# students: 42 + 1

| S17  | S18 | S19 |
|------|-----|-----|
| 13+1 | 10  | 19  |

1.) Multiple Choice

(20 pts.)

For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

- (a) If the net torque on an extended object is zero, the angular velocity of that object is constant.

TRUE

FALSE

- (b) If the center of mass of an extended object is at rest, the object must have zero kinetic energy.

TRUE

FALSE

- (c) When a hoop and a solid cylinder roll down an inclined plane (starting from rest), the hoop will reach the bottom first.

TRUE

FALSE

- (d) Pressure is a scalar quantity.

TRUE

FALSE

- (e) The buoyant force on a fully submerged object only depends on the volume of that object, not on its density.

TRUE

FALSE

| No. | Points |
|-----|--------|
| 1   | HQ     |
| 2   | KD     |
| 3   | RR     |
| 4   | HQ     |
| 5   | SD     |
| Sum |        |

2.) *Translational and Rotational Kinetic Energy*

(20 pts.)

Starting from rest, a solid sphere rolls down an inclined plane without slipping. The vertical height of the plane is 45cm. (The moment of inertia of a solid sphere of radius  $R$  and mass  $m$  for a rotational axis through its center is  $I = \frac{2}{5}mR^2$ ).

- (a) Calculate the translational speed of the sphere at the bottom of the plane.  
 (b) What percentages of the total kinetic energy are in the rotational and in the translational motion of the sphere?

$$\begin{aligned}
 (a) \quad E_o &= E_f \\
 mgh &= \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 & \omega_f &= \frac{v_f}{r} \\
 &= \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{v_f^2}{r^2} & , \quad r &= R \\
 &= \frac{1}{2}mv_f^2 + \frac{1}{5}mv_f^2 \\
 \Rightarrow gh &= \frac{7}{10}v_f^2 & \Rightarrow v_f &= \sqrt{\frac{10}{7}gh} = \boxed{2.51 \frac{m}{s}}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \text{rotational} : \quad \frac{\frac{1}{5}}{\frac{7}{10}} &= \frac{2}{7} = 29\% \\
 \text{translational} : \quad \frac{\frac{1}{2}}{\frac{7}{10}} &= \frac{5}{7} = 71\%
 \end{aligned}$$

### 3.) Torque and Angular Acceleration

(24 pts.)

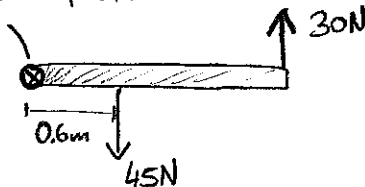
Two persons are pushing on a door as shown below (top view). Assume the door to be a uniform bar of length  $1.4\text{ m}$  and mass  $25\text{ kg}$ , with moment of inertia of  $I = \frac{1}{3}mL^2$  relative to its axis of rotation.

(a) Calculate the net torque on the door.

(b) Calculate the angular acceleration of the door.

(c) When starting from rest, after what time has the door turned through  $45^\circ$ ?

axis of rotation



$$(a) \quad \tau = \tau_1 + \tau_2$$

$$\boxed{\tau_{\text{net}} = -F_1 l_1 + F_2 l_2 = -45 \times 0.6 + 30 \times 1.4 = 15 \text{ Nm}}$$

$$(b) \quad \tau = I\alpha = \frac{1}{3}mL^2\alpha$$

$$\Rightarrow \boxed{\alpha = \frac{\tau}{\frac{1}{3}mL^2} = 0.918 \frac{\text{rad}}{\text{s}^2}}$$

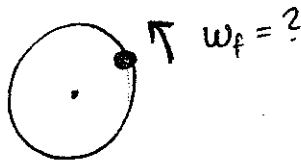
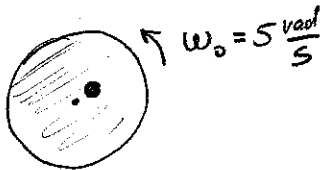
$$(c) \quad \Delta\theta = \frac{1}{2}\alpha t^2 \Rightarrow \boxed{t = \sqrt{\frac{2\Delta\theta}{\alpha}} = 1.31 \text{ s}}$$

$$\Delta\theta = 45 \frac{\pi}{180} = 0.785 \text{ rad}$$

4.) Angular Momentum Conservation

(16 pts.)

A child (mass 25 kg) stands on a merry-go-round (radius 2 m, mass 60 kg), 50 cm from the center. Child and merry-go-round rotate at an initial angular speed of 5 rad/s. The child then moves to the outer rim of the merry-go-round. What is the final (common) angular speed of the child+merry-go-round? (assume the child to be a point particle and the merry-go-round to be a uniform disk with moment of inertia  $I = \frac{1}{2}mr^2$ .)



$$L_o = L_f$$

$$I_o \omega_o = I_f \omega_f$$

$$\left( \frac{1}{2} m r^2 + M r_o^2 \right) \omega_o = \left( \frac{1}{2} m r^2 + M r_f^2 \right) \omega_f$$

$$\Rightarrow \boxed{\omega_f = \frac{(\frac{1}{2} m r^2 + M r_o^2)}{(\frac{1}{2} m r^2 + M r_f^2)} \omega_o = 2.87 \frac{\text{rad}}{\text{s}}}$$

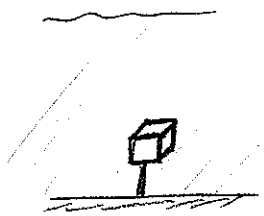
5.) Archimedes Principle

(20 pts.)

A solid uniform plastic cube (with each side of length **25 cm**) is attached to a short massless rope at the bottom of a lake. The rope can support a tension force of up to **70 N** (density of water:  $\rho_w = 1000 \text{ kg/m}^3$ ).

(a) What is the buoyant force on the cube?

(b) What is the minimal density of the cube so that the rope does not break?



$$(a) \quad \textcircled{10} \quad F_B = \rho_w V_{\text{obj}} g = \boxed{153.1 \text{ N}}$$

$$(b) \quad \Sigma F = 0 \quad \textcircled{2}$$

$$\textcircled{3} \quad -T_{\text{max}} + F_B - m_{\text{min}} g = 0$$

$$\textcircled{3} \Rightarrow m_{\text{min}} = \frac{1}{g} (F_B - T_{\text{max}}) = 8.48 \text{ kg}$$

$$\textcircled{2} \Rightarrow \rho_{\text{min}} = \frac{m_{\text{min}}}{V} = \boxed{542.8 \frac{\text{kg}}{\text{m}^3}}$$