Exam-3 Key (Spring 2007)

1.) *Multiple Choice* (*21 pts.*)

For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

(a) When increasing the tension in a string, the wave speed on the string decreases.
TRUE FALSE

(b) When increasing the equilibrium temperature of a black body from $40°F$ to $80°F$, the radiated power of that body increases by a factor 2^4.
TRUE FALSE

(c) The total internal energy of an ideal gas amounts to the sum of the kinetic energies of all molecules in the gas.
TRUE FALSE

(d) At room temperature, the average speed of hydrogen molecules ($m_{H_2} = 2u$) is smaller than the average speed of nitrogen molecules ($m_{N_2} = 28u$).
TRUE FALSE

(e) Spontaneous heat flow increases the entropy of the Universe.
TRUE FALSE

(f) A Carnot engine operates with reversible processes only.
TRUE FALSE

(g) If a gas does adiabatic expansion work, the total entropy of the universe does not increase.
TRUE FALSE

No.	Points
1	WG
2	KR
3	RR
4	JV
5	WG
6	LN
Sum	

V1

2.) *Doppler Effect* (18 pts.)

A police car, with its siren on, is approaching a tunnel (which passes through a mountain) with a speed of $105mph$. At rest, the siren emits a pure tone of frequency $1200Hz$. An observer is standing next to the tunnel entrance. [1400Hz] ($1m/s = 2.25mph$)

(a) What is the frequency of the siren tone that the observer hears?

(b) If the siren emits sound with a power of [2W] $1W$, and the car is $1km$ away from the observer, with what intensity level does the observer hear the siren (assume the sound wave to be a spherical wave)?

(c) The siren tone is reflected from the mountain back to the police officer in the car. With what frequency does the police officer hear the reflected siren sound?

(a) $f_o = f_s \dfrac{1}{1 - V_s/V} = 1389\,Hz$ [1620 Hz]

(b) $I = \dfrac{P}{A} = \dfrac{1W}{4\pi(1000m)^2} = 7.96 * 10^{-8} \dfrac{W}{m^2}$

$\beta = 10dB \log\left(\dfrac{I}{I_o}\right) = 49\,dB$ [52dB]

(c) $f_{off} = f_o \dfrac{1 + V_o/V}{1} = 1578\,Hz$ [1840 Hz]

[0.4kg]
~~[0.7kg]~~

3.) *Calorimetry and Entropy Production* (*18 pts.*)

A thirsty student prepares himself an ice tea by mixing 1.5kg of tea with 0.5kg of ice in a perfectly insulating Aggie cup. The initial temperature of the tea is 30°C, while the initial temperature of the ice is -10°C. Neglect the heat capacity of the cup, and assume the heat capacity of the tea to be equal to that of water. (specific heat capacity of water and ice: $c_{\text{water}} = 4186J/(kg \cdot C^o)$, $c_{\text{ice}} = 2000J/(kg \cdot C^o)$, latent heat of fusion for water: $L_f =$ ~~33400~~ J/kg).

334000

(a) After equilibration, what is the final temperature of the mixture?

(b) What is the amount of net heat transfered from the ice/water to the tea?

(c) Calculate the entropy changes of the tea and of the ice/water (using their average temperatures in the process), as well as the total change in entropy. Is the total consistent with the second law of thermodynamics?

(a) $|Q_{H_2O}| = |Q_{tea}|$

$$m_{H_2O}\left(c_{ice}\,\Delta T_{ice} + L_f + c_w\,\Delta T_w\right) = -m_{tea}\,c_{tea}\,\Delta T_{tea}$$

$$m_{H_2O}\left(c_{ice}\,10C^o + L_f + c_w(T_f - 0)\right) = +m_{tea}\,c_{tea}\,(30 - T_f)$$

$$\Rightarrow \boxed{T_f = \frac{m_{H_2O}(c_{ice}\,10 + L_f) - m_{tea}\,c_w\,30}{-c_w\,(m_{tea} + m_{H_2O})}} = \text{~~19.3~~}\ ^oC \quad [16.4^oC]$$

1.36°C [5.89°C]

(b) $|Q| = m_{tea}\,c_w\,\Delta T_{tea} =$ ~~67 1175~~ 179850 J [~~85422~~ J] 151390

(c) $\bar{T}_{H_2O} = \frac{263.15 + \text{~~292.45~~}}{2} \simeq$ ~~278~~ 269 °K [≃ ~~276~~ °K] 271

$\bar{T}_{tea} = \frac{303.15 + 292.45}{2} \simeq$ ~~298~~ 289 °K [≃ ~~296~~ °K] 291

$$\boxed{\Delta S_{H_2O} = \frac{Q}{\bar{T}_{H_2O}} = +\text{~~241~~}\ 669\ \frac{J}{^oK}} \quad [\text{~~309.5~~}\ J/^oK]\ 559$$

$$\boxed{\Delta S_{tea} = -\frac{Q}{\bar{T}_{tea}} = -\text{~~225~~}\ 622\ \frac{J}{^oK}} \quad [-\text{~~288.6~~}\ J/^oK]\ 520$$

$$\boxed{\Delta S_{tot} = +\text{~~16~~}\ 47\ \frac{J}{^oK}} \geq 0 \ \checkmark \quad [\text{~~21~~}\ J/^oK]\ 39$$

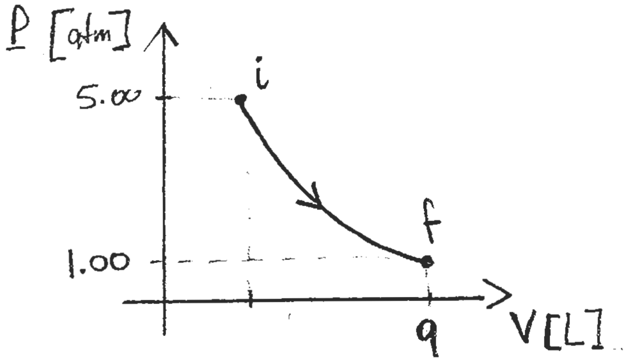
4.) *Adiabatic Expansion* [220J] *(15 pts.)*

In the p-V diagram shown below, 110J of work was done by 0.6 mol of [0.8 mol] an ideal monatomic gas ($\gamma = 5/3$) during an adiabatic process (i.e., $Q = 0$).

(a) By how many joules did the internal energy of the gas change (include the proper plus/minus sign)?

(b) What was the initial volume of the gas?

(c) What are the initial and final temperature of the gas?



(a) $\boxed{\Delta U = Q - W = -110\,J}$ [-220J]

(b) $P_i V_i^{\gamma} = P_f V_f^{\gamma} \Rightarrow \boxed{V_i = V_f \left(\frac{P_f}{P_i}\right)^{1/\gamma} = 3.43\,L}$

(c) $PV = nRT$ $\boxed{T_i = \frac{P_i V_i}{nR} = 348°K}$ [261°K]

$\boxed{T_f = \frac{P_f V_f}{nR} = 183°K}$ [137°K]

It turns out, that using $\Delta U = \frac{3}{2} n R \Delta T$, one finds $\boxed{\Delta U = -1234\,J \; [-1237J]}$ $= -W$

i.e., the problem is overspecified.

Full credit was given either way.

5.) *Carnot Engine* *(15 pts.)*

In each cycle, a power plant extracts $7000J$ of energy from a hot reservoir at a temperature of $250°C$ [150°C] while dumping an unknown amount of heat into the environment which is at a temperature of $25°C$. Assume the heat engine to be a Carnot engine.

(a) What is the thermal efficiency of the engine?

(b) How much mechanical work does the engine perform in each cycle?

(c) How much heat is dumped into the environment in each cycle?

(a) $e = 1 - \frac{T_c}{T_H} = 1 - \frac{298.15}{523.15} = 43\%$ [29.5 %]

(b) $W = eQ_H = 3011\,J$ [2068 J]

(c) $Q_H = Q_c + W$

$\Rightarrow Q_c = Q_H - W = 3989\,J$ [4932 J]

6.) *Ideal Gas Expansion* [600J] (6+7 pts.)

A fixed amount of Helium gas expands slowly doing $400J$ of work in the process (assume Helium to be an ideal monatomic gas). Find the heat added to the gas and the change in internal energy if the expansion proceeds

(a) isothermal, $(T = \text{const})$

(b) isobaric. $(P = \text{const})$

$\Delta T = 0 \quad (T = \text{const})$

(a) $\Rightarrow \boxed{\Delta U = 0} = Q - W$

$\Rightarrow \boxed{Q = W = 400J}$ [600J]

(b) $P = \text{const}$, $W = P\Delta V = 400J$

$\Delta U = Q - W$

$\Delta U = \frac{3}{2} n R \Delta T$, $PV = nRT$

$\Rightarrow \Delta T = \frac{P}{nR} \Delta V$

$= \frac{3}{2} P \Delta V$

$\boxed{\Delta U = 600J}$ [900J]

$\Rightarrow \Delta U = \frac{3}{2} P \Delta V = Q - P \Delta V$

$\Rightarrow \boxed{Q = \frac{5}{2} P \Delta V = \frac{5}{2} P \Delta V = 1000J}$ [1500J]