Exam III Solutions Fall '11

1.) *Multiple Choice* *(18 pts.)*

For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

(a) In non-uniform circular motion, centripetal and tangential acceleration are at 90° relative to each other.

TRUE FALSE

(b) The moment of inertia of the same object depends on the axis about which that object rotates.

TRUE FALSE

(c) If the external forces on a rigid body balance each other, the external torques on that body also add up to zero.

TRUE FALSE

(d) If a solid cylinder and a hollow sphere roll down the same slope without slipping, both starting from rest, the hollow sphere arrives at the bottom first.

TRUE FALSE

(e) Angular momentum is a vector quantity.

TRUE FALSE

(f) In simple harmonic motion, the maximal acceleration acts when the speed of the object is zero.

TRUE FALSE

No.	Points
1	PZ
2	HZ
3	TG
4	HP
5	RR
Sum	

2.) *Torque and Angular Kinematics* *(21 pts.)*

A grindstone in the form of a solid disk with radius 0.7 m and mass 160 kg is rotating about the symmetry axis through its center of mass at 1300 rev/min. You apply a tangential force of 40N to the outer rim of the stone until it comes to a stop.

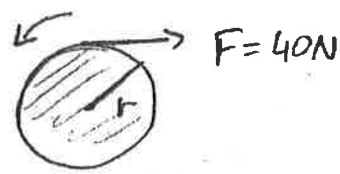
(a) Calculate the angular acceleration of the grindstone during the slow-down.

(b) Calculate how many seconds it takes the grindstone to come to a stop.

(c) How many revolutions did the grindstone go through during the stopping process?

(a) $\tau = I\alpha$

$\tau = Fr$



$Fr = I\alpha$

$I = \frac{1}{2}mr^2$

$$\boxed{\alpha = \frac{Fr}{\frac{1}{2}mr^2} = \frac{2F}{mr} = 0.714\ \frac{rad}{s}}$$

(b) $w = w_0 - \alpha t$

$w = 0$

$w_0 = \frac{1300 * 2\pi}{60s} = 136.14\ rad$

$$\Rightarrow \boxed{t = \frac{w_0}{\alpha} = 191\ s}$$

(c) $\Delta\theta = w_0 t - \frac{1}{2}\alpha t^2 = 12980\ rad$

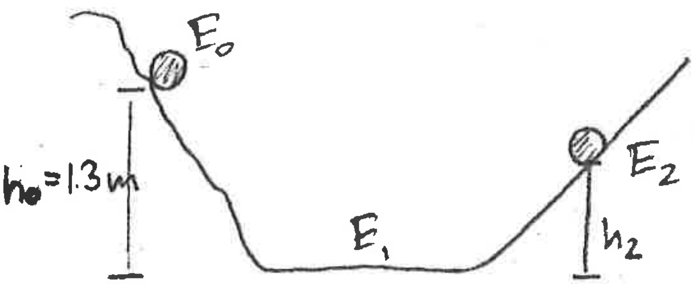
$$\boxed{\#\ rev = \frac{\Delta\theta}{2\pi} = 2066}$$

3.) *Rotational Kinetic Energy* *(20 pts.)*

A solid sphere rolls done a hill on a rough surface (no slipping), starting from rest at a height of 1.3 m above ground. It reaches the valley (ground) and then rolls up a frictionless incline (complete slipping).

(a) Calculate the linear speed of the sphere in the valley.

(b) Calculate the maximal height the sphere reaches when climbing the smooth incline.



(a) $E_0 = E_1 = K_1^{lin} + K_1^{rot}$

$$mgh_0 = \frac{1}{2} m v_1^2 + \frac{1}{2} I \omega_1^2$$

$$\omega_1 = \frac{v_1}{r}$$

$$= \frac{1}{2} m v_1^2 + \frac{1}{2}\frac{2}{5} m r^2 \frac{v_1^2}{r^2}$$

$$I = \frac{2}{5} m r^2$$

$$= \frac{1}{2} m v_1^2 \left(1 + \frac{2}{5}\right) = 0.7\, m v_1^2$$

$$\Rightarrow \boxed{v_1 = \sqrt{\frac{g h_0}{0.7}} = 4.27\,m/s}$$

(b) $E_1 = E_2$

$$K_1^{lin} + K_1^{rot} = mgh_2 + K_2^{rot}$$

$$K_2^{rot} = K_1^{rot} \quad \text{(slipping)}$$

$$\frac{1}{2} m v_1^2 = mgh_2$$

$$\boxed{h_2 = \frac{v_1^2}{2g} = 0.93\,m}$$

4.) *Angular Momentum Conservation* (*20 pts.*)

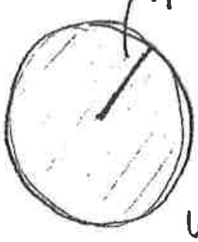
A uniform star of radius $5.5 \times 10^5\ km$ rotates at a rate of one revolution in 25 days. Suddenly, the internal gravitational forces make all of its matter collapse into a uniform "neutron star" of radius $15km$. The mass of the star is 1.4 solar masses ($M = 2.8 * 10^{30}$ kg)

(a) At how many revolutions per second does the neutron star rotate?

(b) Calculate how much work the gravitational collapse did on the star.

(a)

$r_1 = 5.5 * 10^5$ km



$\longrightarrow$ $r_2 = 15$km

$\omega_2 = ?$

$$\omega_1 = \frac{2\pi}{25d}$$

$$= 2\pi / (25 * 24 * 60 * 60) = 2.91 * 10^{-6}\ \frac{rad}{s}$$

$$L_1 = L_2$$

$$I_1 \omega_1 = I_2 \omega_2$$

$$\Rightarrow \omega_2 = \omega_1 \frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}\omega_1 = 3.91 * 10^3\ \frac{rad}{s}$$

$$\boxed{\frac{\#rev}{s} = \frac{\omega_2}{2\pi} = 622\ rev/s}$$

(c)

$$W = \Delta K = K_f - K_i = K_2 - K_1$$

$$= \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2$$

$$\boxed{W = \frac{1}{2} m \frac{2}{5}(r_2^2 \omega_2^2 - r_1^2 \omega_1^2) = 1.9 * 10^{45}\ J}$$

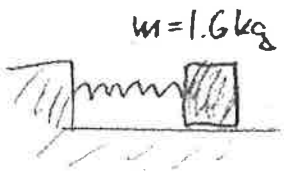
5.) *Simple Harmonic Motion* (21 pts.)

A 1.6 kg block is attached to an ideal spring, performing simple harmonic motion on a frictionless horizontal surface. The force constant of the spring is 260 N/m. Initially, the spring is in its relaxed position, but the block is moving at a speed of 13 m/s. Calculate

(a) the amplitude of the motion.

(b) the maximal acceleration of the block.

(c) the period of the motion.



(a) $E_0 = E_1$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} k A^2 \Rightarrow \boxed{A = v_0 \sqrt{\frac{m}{k}} = 1.02 m}$$

(b) $F_{max} = m a_{max}$

$= k A$

$$\boxed{a_{max} = \frac{kA}{m} = 165.7 \frac{m}{s^2}}$$

(c) $\boxed{T = 2\pi \sqrt{\frac{m}{k}} = 0.49 s}$