Exam 3 Fall 2010 Solution Key

1.) *Multiple Choice* *(18 pts.)*

For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

(a) ~~In circular~~ motion, centripetal and angular acceleration are at 90° relative to each other.
TRUE FALSE

(b) The moment of inertia of an object is independent of the axis about which the object rotates.
TRUE FALSE

(c) ~~Riding a~~ bike is possible thanks to angular momentum conservation.
TRUE FALSE

(d) When a nonzero net force is acting on a rigid object it necessarily implies that there is also a net torque ~~acting on~~ that object.
TRUE FALSE

(e) ~~In simple~~ harmonic motion the maximal speed occurs when the acceleration is zero.
TRUE FALSE

(f) ~~The period~~ of a ~~pendulum becomes~~ smaller if the length of the pendulum is increased.
TRUE FALSE

No.	Points
1	HZ
2	AP
3	HZ
4	RR
5	KH
6	FZ.
Sum	

2.) *Angular Kinematics* (18 pts.)

A potter is done with his work and leaves his turntable (uniform cylinder of mass $250kg$ and radius 0.15m) alone. At this moment the turntable is still spinning at $420rpm$, but is gradually slowing down. It takes 45 revolutions until it stops.

(a) What is the angular acceleration of the turntable in the process?

(b) How long does it take for the turntable to come to a stop after being left alone?

(c) What is the net torque on the turntable in the process?

(a) $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ $\qquad \omega = 0$, $\omega_0 = 2\pi \frac{420}{60} \frac{rad}{s}$

$\theta - \theta_0 = 2\pi * 45 \text{ rad}$

$$\Rightarrow \boxed{\alpha = \frac{\omega^2 - \omega_0^2}{2\Delta\theta} = -3.42 \frac{rad}{s^2}}$$

(b) $\omega = \omega_0 + \alpha t$

$$\Rightarrow t = \frac{\omega - \omega_0}{\alpha} = \frac{-\omega_0}{\alpha} = 12.86 \, s$$

(c) $\tau = I\alpha$ $\qquad I = \frac{1}{2}mr^2$

$$\boxed{\tau = \frac{1}{2} 250 (0.15)^2 (-3.42) = -9.62 \, Nm}$$

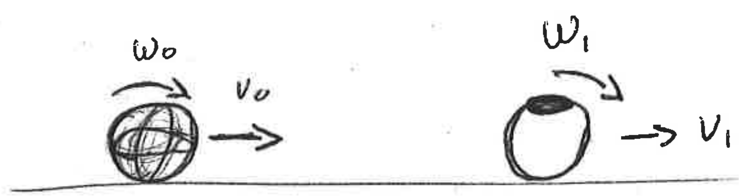
3.) *Angular Momentum Conservation* *(14 pts.)*

A plastic ball (diameter 25cm, mass 200g) is rolling forward on a horizontal surface with a linear speed of 0.8m/s. A piece of mud (mass 150g) drops on top of the ball and sticks to it.

(a) Considering the ball as a hollow sphere, calculate its moment of inertia before and after the mud attaches to it.

(b) Calculate the angular and linear speed of the ball after the mud attaches to it.

(a)



$$\boxed{I_0 = \frac{2}{3} m r^2 = 0.00208\, kgm^2}\ , \quad \boxed{I_1 = \frac{2}{3} m r^2 + m_{mud} r^2} = \boxed{0.00443\, kgm^2}$$

(b)

$$L_0 = L_1$$

$$I_0 \omega_0 = I_1 \omega_1 \quad \Rightarrow \quad \boxed{v_1 = \frac{I_0}{I_1} v_0 = 0.38 \frac{m}{s}}$$

$$\boxed{\omega_1 = v_1/r = 3.04 \frac{rad}{s}}$$

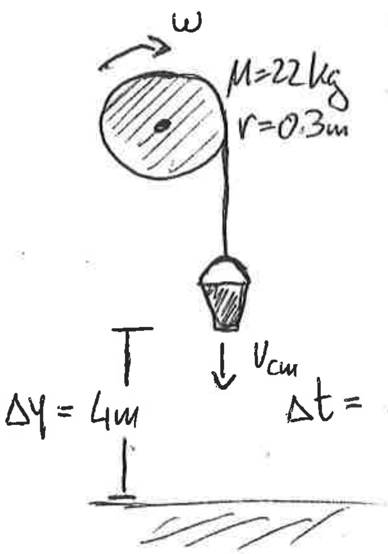
4.) *Mechanical Energy Conservation* radius 30cm (18 pts.)

A bucket of water is attached to a pulley (solid cylinder of mass $22kg$) via an ideal rope. The system is released from rest. The bucket, originally at $4m$ above the ground, drops to the ground within $1.8s$, splashing most of the water.

(a) Assuming a constant acceleration, how large are the linear acceleration of the bucket and the angular acceleration of the pulley during the falling process? Express your result for the bucket in percentage of $g = 9.8m/s^2$.

(b) What is the total mass of bucket plus water before hitting the ground?

(c) What is the tension in the rope during the fall?



(a) $\Delta y = y - y_0 = +\frac{1}{2} a_{cm} t^2$

$$\boxed{a_{cm}} = \frac{2\Delta y}{t^2} = \frac{-8}{(1.8)^2} = -2.47 \frac{m}{s^2} = \boxed{-25\% \, g}$$

$$\alpha = \frac{a_{cm}}{r} = -8.23 \frac{rad}{s}$$

(b) pulley: $\tau_{net} = I\alpha = -Tr \Rightarrow T = \frac{I\alpha}{r}$ (1)

bucket: $F_{net} = m a_{cm} = (-mg + T)$

$\Rightarrow m(a_{cm} + g) = T$ (2)

$m = T/(a_{cm} + g)$

(1) into (2): $\boxed{m = \frac{I\alpha}{r(a_{cm} + g)} = \frac{\frac{1}{2} M r^2 \alpha}{r(a_{cm} + g)} = 3.7 \, kg}$

(c) $\boxed{T = m(g + a_{cm}) = m \, 0.75 g = 27.2 \, N}$

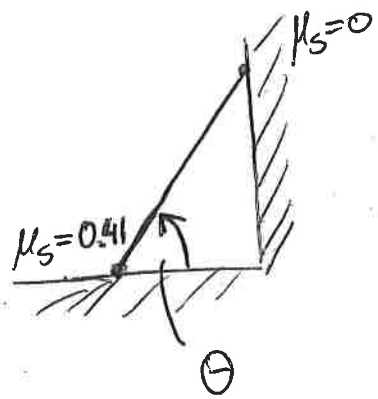
5.) *Ladder in Equilibrium* *(18 pts.)*

A ladder of unknown mass m and length L is leaning against a vertical wall, at an angle Θ between the ladder and the horizontal ground (see figure below). The static friction coefficient between ladder and ground is 0.41, and zero between the ladder and the smooth wall.

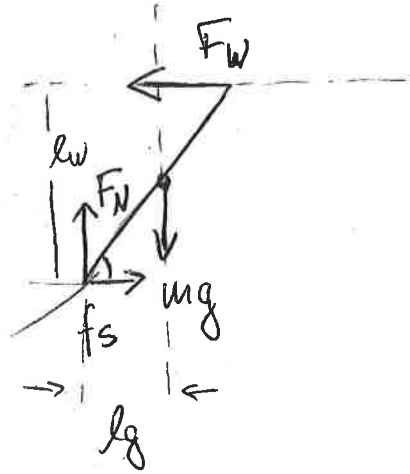
(a) Draw a free body diagram of the ladder.

(b) Formulate force equilibrium and torque equilibrium and work out the lever arms in terms of the ladder length (L) and mass (m).

(c) Calculate the smallest angle, Θ_{min}, before the ladder starts sliding.



(a)



(b) $0 = \sum_i F_x = f_s - F_W$

$0 = \sum_i F_y = F_N - mg$

$0 = \sum_i \tau = F_W \, l_W - mg \, l_g$ with: $l_W = L \sin\theta = L \cos(90 - \theta)$

$l_g = \frac{L}{2} \cos\theta$

(c) $\Theta_{min} \rightarrow f_s = f_s^{max} = \mu_s F_N$

$F_W = f_s^{max} = \mu_s F_N$, $F_N = mg$

$= \mu_s mg$

$\Rightarrow 0 = \mu_s \quad L \sin\Theta_{min} - mg \frac{L}{2} \cos\Theta_{min}$

$0 = \mu_s \sin\Theta_{min} - \frac{1}{2} \cos\Theta_{min} \Rightarrow \frac{1}{\tan\Theta_{min}} = 2\mu_s$

$\Rightarrow \boxed{\Theta_{min} = \tan^{-1}(1/2\mu_s) = 50.65^\circ}$

6.) *Simple Harmonic Motion* (15pts.)

A block of mass 1.2kg is moving on a horizontal frictionless surface and attached to a horizontal spring with force constant 300N/m. The block is performing simple harmonic motion with an amplitude of 0.6m.

(a) Calculate the frequency of the periodic motion.

(b) Calculate the maximal speed during the motion.

(c) Calculate the total mechanical energy in the motion.

(a) $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}} = \boxed{2.52\ Hz}$

(b) $\frac{1}{2} m v_{max}^2 = \frac{1}{2} k A^2$

$\Rightarrow \boxed{v_{max}} = \omega A = \boxed{9.5\ \frac{m}{s}}$

(c) $\boxed{E_{tot}} = K_{max} = \frac{1}{2} m v_{max}^2 = \boxed{54\ J}$