Exam-3 Solutions (Fall 2007)

1.) *Multiple Choice* *(18 pts.)*

For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

(a) If an object is ~~in a circular~~ motion, a tangential acceleration changes the angular speed of that object.
TRUE FALSE

(b) When a hoop ($I = MR^2$) and a solid cylinder ($I = 0.5MR^2$) of identical mass and radius roll down the same inclined plane (starting from rest), the hoop will reach the bottom first.
TRUE FALSE

(c) The total kinetic energy of an extended object is the difference between its linear and rotational energy.
TRUE FALSE

(d) ~~If a nonzero~~ net torque acts on an object, the angular velocity of that object changes.
TRUE FALSE

(e) If there is no net torque acting on a system, the angular momentum of that system is ~~conserved~~.
TRUE FALSE

(f) The time dependence of the velocity in a Simple Harmonic Motion follows a sinusoidal ~~behavior~~.
TRUE FALSE

No.	Points
1	SM
2	SR
3	MP
4	KK
5	RR
6	MS
Sum	

2.) *Angular Kinematics* *(14 pts.)*

A fan having a constant angular acceleration of $4.5 rad/s^2$ requires 6.5s to rotate through 125rad. [3.5s]

(a) What is the average angular velocity of the fan during the 6.5s interval? [3.5]

(b) What are the initial and final angular velocity for the 6.5s interval? [3.5]

(a) $\boxed{\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{125\,rad}{6.5s} = 19.23\ \frac{rad}{s}}$ [35.71 $\frac{rad}{s}$]

(b) $\omega_f = \omega_0 + \alpha t$

$\bar{\omega} = \frac{1}{2}(\omega_0 + \omega_f) = \frac{1}{2}(\omega_0 + \omega_0 + \alpha t) = \omega_0 + \frac{1}{2}\alpha t$

$\Rightarrow \boxed{\omega_0 = \bar{\omega} - \frac{1}{2}\alpha t = 4.605\ \frac{rad}{s}}$ [27.84 $\frac{rad}{s}$]

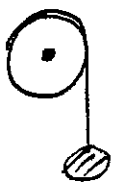
$\boxed{\omega_f = \omega_0 + \alpha t = 33.9\ rad/s}$ [43.59 $\frac{rad}{s}$]

3.) *Rotational Kinetic Energy* and Torque *(16 pts.)*

A massless, unstretchable string is wrapped around a disk (diameter $16cm$) which can rotate without friction about a fixed axis through its center. A stone (mass $750g$) is attached to the end of the string and released from rest. The stone reaches a speed of $3.2m/s$ after having fallen $1.9m$. [1.1m] (the moment of inertia of a solid disk is $I = 0.5MR^2$)

(a) What is the mass of the cylinder? (*hint: energy conservation*)

(b) What is the torque on the disk?



(a) $E_0 = E_f$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad , \omega = \frac{v}{R}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2}$$

$$= \frac{1}{2}\left(m + \frac{1}{2}M\right)v^2$$

$$\Rightarrow \left(\frac{2mgh}{v^2} - m\right)2 = M$$

$$\boxed{M = 3.98\,kg} \quad [1.66\,kg]$$

(b) stone: $\Sigma F = T - mg = ma$

disk: $\Sigma \tau = -TR = I\alpha \qquad \alpha = \frac{a}{R}$

$$\Rightarrow T = -\frac{1}{2}MR^2\frac{a}{R^2} = -\frac{1}{2}Ma$$

$$\Rightarrow \text{stone: } -\frac{1}{2}Ma - mg = ma \quad \Rightarrow \left(\frac{1}{2}M + m\right)a = -mg$$

$$a = mg/\left(\tfrac{1}{2}M - m\right)$$

$$\tau_{net} = I\alpha = -\frac{1}{2}MR^2\frac{1}{R}\,mg/\left(\tfrac{1}{2}M + m\right)$$

$$\boxed{\tau_{net} = -\frac{MmRg}{(M+2m)} = -0.426\,Nm} \quad [-0.309\,Nm]$$

4.) *Rotational Work and Power* [1.6m] *(20 pts.)*

A merry-go-around (a uniform disk of radius $2.4m$ and mass $85kg$) can turn around an axis through its center without friction. A child applies a tangential force of $30N$ to the outer rim of the merry-go-round (initially at rest) for a duration of $9s$. (the moment of inertia of a uniform disk is $I = 0.5MR^2$)

(a) What is the torque on the merry-go-round?

(b) What is the final angular velocity of the merry-go-round?

(c) How much work did the child do on the merry-go-round?

(d) What average power did the child supply?

(a) $\tau = F\ell = 72\ Nm$ ✓ [48 Nm] ✓

(b) $\omega_f = \omega_0 + \alpha t = 2.65\ \frac{rad}{s}$ ✓ [3.97 $\frac{rad}{s}$ ✓]

$\alpha = \frac{\tau}{I} = \frac{\tau}{\frac{1}{2}MR^2} = 0.294\ \frac{rad}{s^2}$

[0.441 $\frac{rad}{s^2}$]

(c) $W = \tau\theta = 858\ J$ ✓ [858 J] ✓

$\theta = \frac{1}{2}\alpha t^2 = 11.91\ rad$ [17.87 rad]

(d) $\bar{P} = \tau\bar{\omega} = 72\ \frac{2.65}{2} = 95.3\ W$ ✓ [95.3 W] ✓

5.) *Elastic Deformation* *(16 pts.)*

A $400kg$ load is attached to the end of a steel cable suspended from a crane. The cable has an unstretched length of $5m$ and a cross sectional area of $0.75cm^2$. ~~The load swings back and forth, with a maximal speed of $0.7m/s$ at the lowest point of the motion.~~ [0.45cm²] (the Young's modulus of steel is $2 \cdot 10^{11}$ Pa.)

(a) Calculate the stress in the cable.

(b) By how much does the wire stretch?

(a) $\text{stress} = \frac{F_\perp}{A} = \frac{T}{A}$ $\quad -mg + T =$

$\Rightarrow T = mg = 3920N$

$\text{stress} = \frac{3920}{(0.75) \cdot 10^{-4} m^2} = 5.23 \cdot 10^7 Pa$ [$8.71 \cdot 10^7 Pa$]

(b) $Y = \frac{\text{stress}}{\Delta l / l_0} \Rightarrow \Delta l = \frac{\text{stress} \; l_0}{Y} = 0.0013 \, m$ [0.0022m]

6.) *Energy in Simple Harmonic Motion* *(16 pts.)*

A 1400g [2400g] block, moving on a horizontal frictionless surface, is attached to a horizontal ideal spring with force constant $250 N/m$. The block has an initial ~~speed~~ velocity of $-1.5 m/s$ at a position of $+0.25m$ relative to the position where the spring is relaxed.

(a) Calculate the amplitude of the motion.

(b) Calculate the maximal acceleration of the motion.

(a) $E_0 = E_1$

$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

$$\boxed{A = \left(\frac{kx^2 + mv^2}{k}\right)^{1/2} = \left(x^2 + \frac{m}{k}v^2\right)^{1/2} = 0.274\,m} \quad [0.29m]$$

(b) $F_{max} = m a_{max}$

$$\boxed{a_{max} = \frac{F_{max}}{m} = \frac{kA}{m} = 48.9\,\frac{m}{s^2}} \quad \left[30.2\,\frac{m}{s^2}\right]$$