Exam-2 Spring '15 (v1/2)

1.) *Multiple Choice* (*18 pts.*)

For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

(a) If a car takes a turn on a road without sliding, the static friction force between tires and road acts as a centripetal force.
TRUE FALSE

(b) Work is a vector quantity.
TRUE FALSE

(c) The tension force is a conservative force.
TRUE FALSE

(d) If a negative net work is done on an object, the object's speed is decreasing.
TRUE FALSE

(e) In a 2-dimensional collision without external forces, the x- and y-component of the total momentum are conserved separately.
TRUE FALSE

(f) If no external forces act on a 2-body system, its center-of-mass velocity does not change.
TRUE FALSE

No.	Points
1	ZW
2	CH
3	RR
4	LS
5	RL
Sum	

2.) *Satellite Motion* (*20 pts.*)

Assume the Moon (mass M_M=7.35·$10^{22}kg$) to be in a circular orbit around Earth (mass M_E=6·$10^{24}kg$) at a radius of 384000km.

(a) What is the gravitational force exerted by the Earth on the Moon?

(b) Calculate the period of the Moon's revolution around Earth (in days).

(a) $$F_G = G\frac{m_1 m_2}{r^2} = 6.67\cdot 10^{-11}\,\frac{7.35\cdot 10^{22}\cdot 6\cdot 10^{24}}{(3.84\cdot 10^{8})^2}$$

$$\boxed{F_G = 2.0\cdot 10^{20}\,N}$$

$$\boxed{a = \frac{F_G}{M_M} = 0.0027\,\frac{m}{s^2}}$$

(b) $$V = \frac{2\pi R}{T} \Rightarrow T = \frac{2\pi R}{V} \quad \text{need } v$$

$$\frac{mv^2}{R} = G\frac{mM_E}{R^2} \Rightarrow v = \sqrt{\frac{GM_E}{R}} = 1020\,\frac{m}{s}$$

$$\Rightarrow \boxed{T = \frac{2\pi\, 3.84\cdot 10^8}{1020} = 2.36\cdot 10^6\,s \simeq 27\text{ days}}$$

3.) *Vertical Circular Motion and Energy Conservation* *(22 pts.)*

Consider a frictionless roller coaster with a decline followed by a loop-the-loop (radius $4\,m$), as sketched below. (6m)

(a) What is the minimal speed required for the car at the top of the loop, in order not to fall off the track? *(hint: the normal force is zero in this case)*

(b) How high above the ground should the car start out on the initial decline to just make it through the loop without falling off the track?

(a) $F_c \geq F_G$

$$\frac{mv^2}{r} \geq mg$$

$$\boxed{v \geq \sqrt{gr} = 6.26\ \frac{m}{s}} \quad (7.67\ \tfrac{m}{s})$$

(b) Mechanical Energy Conservation:

$$E_0 = E_1$$

$$mgh_0 = \frac{1}{2}mv_1^2 + mg(2r) \qquad v_1 = \sqrt{gr}$$

$$\Rightarrow \boxed{h_0 = \frac{gr}{2g} + 2r = 2.5r = 10\,m} \quad (15m)$$

4.) *Nonconservative Work* (45m) (20 pts.)
A box is sliding on a horizontal surface for **25** m, causing its speed to decrease from initially 15 m/s to 6 m/s.

(a) Calculate the kinetic friction coefficient between the box and the surface.

(b) How many additional meters does the box slide before coming to a stop?

(a) $W_{nc} = \Delta K$

$$-\mu_K F_N \Delta x = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$\mu_K m g \Delta x = \frac{1}{2} m (v_i^2 - v_f^2)$$

$$\Rightarrow \quad \mu_K = \frac{v_i^2 - v_f^2}{2 g \Delta x} = 0.39 \quad (0.21)$$

(b) $W_{nc} = \Delta K = K_f - K_i = -\frac{1}{2} m v_i^2$

$$-\mu_K m g \Delta x_2 = -\frac{1}{2} m v_i^2$$

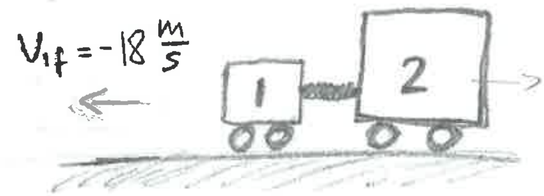
$$\Rightarrow \quad \boxed{\Delta x_2 = \frac{v_i^2}{2 g \mu_K} = 4.76\,m} \quad (8.57m)$$

5.) *Momentum Conservation* (30cm) (20 pts.)

A massless spring of unknown spring constant is compressed by $22\,cm$ in between two carts of masses $m_1 = 40\,kg$ and $m_2 =$ (200) $600\,kg$, both being originally at rest. The system is released and the two carts recede from each other, leaving the spring behind on the ground. Cart 1 reaches a final velcocity of $-18\,m/s$, in negative x-direction. Neglect friction.

(a) What is the final velocity of cart 2?

(b) What is the spring constant?



(a) $P_i = P_f$

$0 = m_1 V_{1f} + m_2 V_{2f}$

$\Rightarrow \boxed{V_{2f} = -\frac{m_1}{m_2} V_{1f} = 1.2\,\frac{m}{s}}$ $\left(3.6\,\frac{m}{s}\right)$

(b) $E_i = E_f$

$\frac{1}{2} k x^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$

$\Rightarrow k = \frac{1}{x^2}\left(m_1 V_{1f}^2 + m_2 V_{2f}^2\right) = 2.9 \cdot 10^5\,\frac{N}{m}$ $\left(1.7 \cdot 10^5\,\frac{N}{m}\right)$