Solution Key Exam-2 S07

1.) *Multiple Choice* (18 pts.)

For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

(a) If only conservative forces are acting, kinetic energy is conserved.
TRUE FALSE

(b) When a stone is launched from the edge of a cliff with a given initial speed, its final speed when hitting the bottom does not depend on the launch angle (neglect friction and air drag).
TRUE FALSE

(c) If the net work applied to a moving object is negative, the speed of the object must decrease.
TRUE FALSE

(d) If there is no net torque acting on a rigid rotator, its angular speed must be zero.
TRUE FALSE

(e) The moment of inertia of a rotating object is a vector quantity.
TRUE FALSE

(f) Angular momentum is a vector quantity.
TRUE FALSE

No.	Points
1	
2	
3	
4	
5	
6	
Sum	

$9.8 \frac{m}{s}$

2.) *Energy Conservation and Non-Conservative Work* (18 pts.)
A skier is sliding on a frictionless horizontal snow surface at a speed of $22mph$. He then encounters a rough horizontal ice patch of length $4.3m$, [6.3] and, after that, slides down an incline of height $8.5m$ (no friction on the incline). The kinetic friction coefficient between the skies and the rough ice is 0.35; neglect air drag. ($1m/s$=$2.25mph$)

(a) What is the speed of the skier right after crossing the ice patch?

(b) What is the speed of the skier at the bottom of the hill?

(c) If the mass of the skier is $78kg$, how much non-conservative work has been done on the ice patch?

(a) $W_{nc} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$W_{nc} = -f_k \Delta s = -\mu_k F_N \Delta s = -\mu_k mg \Delta s$

$\Rightarrow -\mu_k g \Delta s = \frac{1}{2}(v_f^2 - v_i^2)$

$\Rightarrow v_f^2 = v_i^2 - 2\mu_k g \Delta s \Rightarrow \boxed{v_f = 8.13 \frac{m}{s}} = 18.3 mph$

$[7.24 \, m/s]$

(b) $\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgh$

$\Rightarrow \boxed{v_f = \sqrt{v_i^2 + 2gh}} = 15.3 \frac{m}{s}$ $[14.8 \frac{m}{s}]$

(c) $\boxed{W_{nc} = -\mu_k mg \Delta s = -1150J}$ $[-1686 J]$

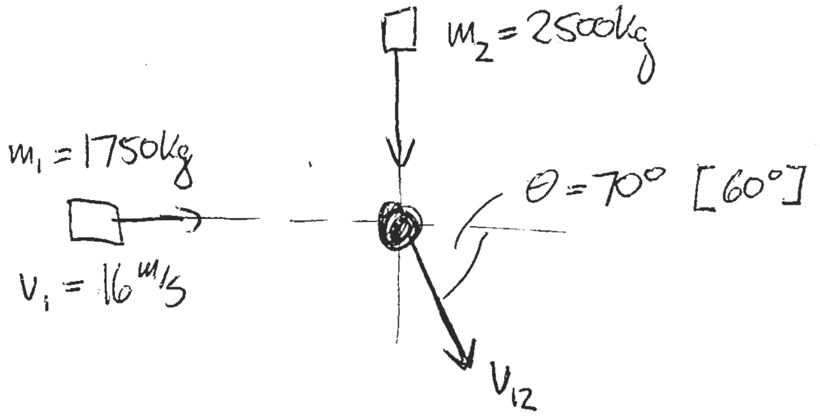
3.) *Inelastic Collision* (18 pts.)

Car 1 (mass m_1=1750kg) and **car 2** (mass m_2=2500kg) are colliding at an intersection. Before the collision, **car 1** was going due east at a speed of 36mph (as retrieved from the board computer), while **car 2** was moving due south. After the collision, both cars are stuck together, leaving the collision point at an angle of 70$degrees$ [60] south of east.

(a) What was the speed of the stuck-together vehicles right after the collision?

(b) What was the speed of **car 2** before the collision? Did it violate the speed limit of 40mph?

(c) How much energy went into the deformation of the vehicles?



(a) $\vec{P}_i = \vec{P}_f$ x-dir.: $m_1 v_1 = M v_{12} \cos\theta$

$\Rightarrow \boxed{v_{12} = v_1 \frac{m_1}{(m_1+m_2)} \frac{1}{\cos\theta} = 19.26 \frac{m}{s} = 43.3\ mph}$ [$13.18 \frac{m}{s}$]

(b) y-dir.: $m_2 v_2 = M v_{12} \sin\theta$

$\boxed{v_2 = \frac{M}{m_2} \sin\theta \cdot v_{12} = 30.8 \frac{m}{s} = 69.2\ mph}$ [43.6 mph]

[$19.4 \frac{m}{s}$] violates speed limit

(c) $\boxed{W_{nc}} = \Delta K = \frac{1}{2} M v_{12}^2 - \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2) = \boxed{-6.2 * 10^5 J}$

[$-3.25 * 10^5 J$]

4.) *Simple Harmonic Motion* (18 pts.)

A $3kg$ block, moving on a frictionless horizontal surface, is attached to an ideal spring with force constant $420N/m$. [340] Initially, the block has a speed of $3.5m/s$ at a displacement of $-0.3m$ away from the equilibrium position of the spring.

6 (a) Calculate the amplitude of the motion.

6 (b) Calculate the maximal force on the block during the motion.

6 (c) Calculate the period of the motion.

(a) $E_1 = E_2$ $\qquad \frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}kA^2$

$$\Rightarrow \boxed{A = \sqrt{x_1^2 + \frac{m}{k}v_1^2} = 0.42m} \qquad [0.46m]$$

(b) $$\boxed{F_{max} = kA = 177\ N} \qquad [142N]$$

(c) $$\omega = \sqrt{\frac{k}{m}} = 2\pi f = 11.83\ rad/s \quad [10.17\ rad/s]$$

$$\Rightarrow \boxed{T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 0.53s} \qquad [0.62s]$$

5.) *Rotational, Translational and Potential Energy* (14pts.)

Starting from rest at an initial height of 55cm [75], a solid steel ball rolls down a hill without slipping. After reaching the bottom, it climbs up another hill with frictionless surface where its spinning motion does not change. Neglect any friction losses:

(a) What is the speed of the ball's center of mass on the bottom of the hill?

(b) To what vertical height above the bottom point does the ball rise?

(a) $E_0 = E_1$

$$mgh_0 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 \qquad I = \frac{2}{5}mR^2$$

$$\omega_1 = \frac{v_1}{R}$$

$$gh_0 = \frac{1}{2}v_1^2 + \frac{1}{2}\frac{2}{5}R^2\frac{v_1^2}{R^2}$$

$$= \frac{1}{2}v_1^2 + \frac{1}{2}\frac{2}{5}v_1^2 = 0.7\,v_1^2$$

$$\Rightarrow \boxed{v_1 = \sqrt{\frac{gh_0}{0.7}} = 2.8\,\frac{m}{s}} \quad [3.24\,\frac{m}{s}]$$

(b) $mgh_f = \frac{1}{2}mv_1^2$

$$\boxed{h_f = v_1^2/2g = 39\,cm} \quad [54\,cm]$$

or 40 cm

6.) *Torque and Angular Acceleration* (14pts.)

A hunter runs out of ammunition and is chased by a bear. The hunter runs through the open door of his mountain hut. The door, which has a mass of $20kg$ and is $1.2m$ wide, must be turned through $90 degrees$ to be closed. The hunter applies a constant torque of $190Nm$ to the door. [120 Nm]

(a) What is the angular acceleration of the door? (Treat the door as a thin rod.)

(b) How long does it take for the door to close?

(a) $\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I}$, $I = \frac{1}{3}ML^2$

$\Rightarrow \boxed{\alpha = 19.8 \frac{rad}{s^2}}$ $[12.5 \frac{rad}{s^2}]$

(b) $\theta(t) = \frac{1}{2}\alpha t^2 \Rightarrow \boxed{t = \sqrt{\frac{2\theta}{\alpha}} = 0.40s}$ $[0.50s]$