1.) Multiple Choice

For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

(a) In uniform circular motion, the centripetal force is directed parallel to the velocity of the object.
   TRUE  FALSE

(b) Work is a vector quantity.
   TRUE  FALSE

(c) The normal force is a conservative force.
   TRUE  FALSE

(d) The elastic spring force is a conservative force.
   TRUE  FALSE

(e) The momentum of an object points in the same direction as its velocity.
   TRUE  FALSE

(f) Gravitational mass and inertial mass, figuring into Newton's law of gravitation and into
     Newton's 2. law of motion, respectively, are two different concepts.
   TRUE  FALSE

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<thead>
<tr>
<th>No.</th>
<th>Points</th>
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<tr>
<td>1</td>
<td>KH</td>
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<tr>
<td>2</td>
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<td>4</td>
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<td>5</td>
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<td>HZ</td>
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2.) Satellite Motion

The secret service plans to put a surveillance satellite (mass 250 tons) into a circular orbit so that it revolves around Earth 3 times a day. (mass of Earth: \( M_E = 6 \cdot 10^{24} \text{kg} \), radius of Earth: \( R_E = 6400 \text{km} \), 1 ton=1000 kg).

(a) At what height above the Earth surface does the satellite have to be released?

(b) What is the speed of the satellite in its orbit?

(c) What is the centripetal force on the satellite and where does it come from?

\[ F_G = F_c \]
\[ G \frac{m M_E}{r^2} = \frac{m v^2}{r} \]
\[ v = \frac{2 \pi r}{T} \]

\[ \Rightarrow \]
\[ G \frac{M_E}{r^2} = \frac{1}{r} \left( \frac{2 \pi r}{T} \right)^2 = \frac{4 \pi^2 r}{T^2} \]
\[ \Rightarrow \]
\[ GM_E \frac{T^2}{4 \pi^2} = r^3 \]
\[ r = \left( \frac{GM_E T^2}{4 \pi^2} \right)^{\frac{1}{3}} \]

\[ T = \frac{1}{3} \text{ d} = \frac{1}{3} (60 \times 60 \times 24) = 28800 \text{ s} \]

\[ \Rightarrow \]
\[ r = 2.03 \cdot 10^7 \text{m} = 2.03 \times 10^4 \text{km} = 20300 \text{km} \]

\[ H = r - R_E = \boxed{13900 \text{ km}} \]

(b) \[ \boxed{v = \frac{2 \pi r}{T} = 4.44 \text{ km/s}} \]

(c) \[ \boxed{F_c = \frac{m v^2}{r} = 2.42 \times 10^5 \text{ N}} \]

\[ = F_G \text{ gravitational force} \]
3. Energy Conservation in Projectile Motion

A kid throws a stone with initial speed of 28 mph from the edge of a cliff toward the open sea. The cliff is 23 m above sea level. During flight, the stone reaches a maximal height of 7 m above the cliff.

(1 m/s = 2.25 mph)

(a) Calculate the minimal speed of the stone during flight.

(b) Calculate the maximal speed of the stone during flight.

\[
\begin{align*}
V_0 &= 12.44 \text{ m} \\
h_0 &= 23 \text{ m} \\
h_1 &= 35 \text{ m} \\
h_2 &= 0 \\
V_2 &= V_{\text{max}} \\
V_1 &= V_{\text{min}}
\end{align*}
\]

(a) Minimal speed \( V_1 \) at top:

\[
E_0 = E_1
\]

\[
\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_1^2 + mgh_1
\]

\[
\Rightarrow V_1 = \sqrt{v_0^2 + 2g(h_0 - h_1)} = 4.2 \text{ m/s}
\]

(b) Maximal speed at bottom:

\[
V_2 = \sqrt{v_0^2 + 2g(h_0 - h_2)} = 24.6 \text{ m/s}
\]
4.) Power and Uphill Acceleration

A sports car \((m = 1.3 \text{tons})\) accelerates from 0 to 60\(\text{mph}\) in 6.8s on a horizontal road. \((1\text{m/s} = 2.25\text{mph}, \ 1 \text{ton} = 1000 \text{kg})\)

(a) How much net work is done on the car in the process?

(b) What average power is being applied to the car?

(c) In a different run (with the same car), the acceleration process is repeated on an uphill road. The car covers a vertical height difference of 25m. How much longer does it take to go from 0-60?

\[
\begin{align*}
\text{(a)} \quad W_{\text{net}} &= \Delta K = K_f - K_i = \frac{1}{2} m v_f^2 = 4.62 \times 10^5 \text{ J} \\
\text{(b)} \quad P &= \frac{W}{\Delta t} = 6.797 \times 10^4 = 91.15 \text{ hp} \\
\text{(c)} \quad W_{\text{net}} &= \Delta K + \Delta U = \frac{1}{2} m v_f^2 + mgh = 7.807 \times 10^5 \text{ J} \\
\Rightarrow \quad \Delta t &= \frac{W_{\text{net}}}{P} = 11.5 \text{ s} \quad \text{i.e. 4.7s longer}
\end{align*}
\]
5.) **Ballistic Sliding Block**

A cannon from the middle ages is shot at a giant wood block (mass \(0.75\) tons). The projectile, a spherical rock of mass \(80\) kg, hits the block in horizontal direction and gets stuck in it. The block recoils and slides on a horizontal gravel surface for \(63\) m before coming to rest. The kinetic friction coefficient between block and gravel surface is 0.18.

(a) What was the speed of the block+rock right after the rock got stuck?

(b) What was the speed of the rock just before it hit the wood block?

\[ \begin{align*}
\text{(a)} & \\
W_{\text{ke}} &= \Delta K \\
-f_k s &= K_f - K_i = -K_i = - \frac{1}{2} (m_B + m_R) v_i^2 \\
\Rightarrow v_i^2 &= \frac{2f_k s}{(m_B + m_R)} = \frac{2 \mu_k s (m_B + m_R) g}{(m_B + m_R)} = 2 \mu_k s g \\
\sqrt{v_i} &= \sqrt{2 \mu_k g s} = 4.71 \text{ m/s} \\
\text{(b)} & \\
W_R v_R &= (m_B + m_R) v_i \\
\Rightarrow v_R &= \frac{(m_B + m_R)}{m_R} v_i = 48.9 \text{ m/s} 
\end{align*} \]
6.) **Projectile Break-up in 2 Dimensions** (16 pts.)

A middle-aged stone shell ($m = 24 \text{ kg}$), filled with explosive powder (negligible mass), is traveling horizontally due East at a speed of $32 \text{ m/s}$ when it suddenly breaks up into two fragments of mass $m_1 = 10 \text{ kg}$ and $m_2 = 14 \text{ kg}$. Right after break-up both fragments are still traveling in the horizontal plane, with the lighter one heading $40^\circ$ North of East with a speed of $32 \text{ m/s}$. Ignore gravity in this problem.

(a) What is the speed of the heavier fragment right after break-up?

*Hint: you need to keep track of both x and y components*

(b) Calculate the total kinetic energies before and after break-up and then decide whether the powder ignited ($W_{nc} > 0$) or not ($W_{nc} < 0$).

\[ V_0 = 32 \text{ m/s} \]

\[ m = 24 \text{ kg} \]

\[ m_1 \quad 40^\circ \]

\[ m_2 \]

\[ V = \begin{bmatrix} V_{1x} \cos 40^\circ \\ V_{1y} \sin 40^\circ \end{bmatrix} \]

\[ \Rightarrow \]

\[ \begin{align*}
M v_{0x} &= m_1 v_{1x} + m_2 v_{2x} \\
V_{1x} &= V_0 \\
v_{0x} &= 39.54 \text{ m/s} \\
&= \frac{1}{m_2} (M v_{0x} - m_1 v_{1x})
\end{align*} \]

\[ 0 = m_1 v_{1y} + m_2 v_{2y} \Rightarrow v_{2y} = -\frac{m_1}{m_2} v_{1y}, \quad v_{1y} = v_1 \sin 40^\circ \]

\[ v_{2y} = -12.86 \text{ m/s} \]

\[ \Rightarrow V_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = 41.57 \text{ m/s} \]

\[ W_{nc} = \Delta K = K_f - K_i \]

\[ = \left( \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \right) - \frac{1}{2} M v_0^2 = \frac{3730}{8} \text{ J} \]

Positive, i.e., the powder did explode!