Exam II Solution Key (Fall '10)

1.) *Multiple Choice* *(18 pts.)*

For each statement below, circle the correct answer (TRUE or FALSE, no reasoning required).

(a) In uniform circular motion, the centripetal force is directed parallel to the velocity of the object.
TRUE [✓ FALSE]

(b) Work is a vector quantity.
TRUE [✓ FALSE]

(c) The normal force is a conservative force.
TRUE [✓ FALSE]

(d) The elastic spring force is a conservative force.
[✓ TRUE] FALSE

(e) The momentum of an object points in the same direction as its velocity.
[✓ TRUE] FALSE

(f) Gravitational mass and inertial mass, figuring into Newton's law of gravitation and into Newton's 2. law of motion, respectively, are two different concepts.
[✓ TRUE] FALSE

No.	Points
1	KH
2	RR
3	FZ
4	AP
5	YC
6	HZ
Sum	

2.) *Satellite Motion* *(18 pts.)*

The secret service plans to put a surveillance satellite (mass 250 *tons*) into a circular orbit so that it revolves around Earth 3 times a day. (mass of Earth: $M_E = 6 \cdot 10^{24} kg$, radius of Earth: $R_E = 6400 km$, $1\, ton$=1000 kg).

(a) At what height above the Earth surface does the satellite have to be released?

(b) What is the speed of the satellite in its orbit?

(c) What is the centripetal force on the satellite and where does it come from?

(a) $F_G = F_c$

$$G \frac{m M_E}{r^2} = \frac{m v^2}{r} \qquad v = \frac{2\pi r}{T}$$

$$\Rightarrow \frac{G M_E}{r^2} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2} \qquad \Big| \; {*r^2} \atop {*T^2/4\pi^2}$$

$$G M_E \frac{T^2}{4\pi^2} = r^3 \qquad \Rightarrow \quad r = \left(G M_E T^2 / 4\pi^2\right)^{1/3}$$

$$T = \frac{1}{3} d = \frac{1}{3}(60 * 60 * 24) = 28800 \, s$$

$$\Rightarrow \quad r = 2.03 * 10^7 m = 2.03 * 10^4 km = 20300 \, km$$

$$\Rightarrow \quad \boxed{H = r - R_E = 13900 \, km}$$

(b) $\boxed{v = \frac{2\pi r}{T} = 4.44 \, km/s}$

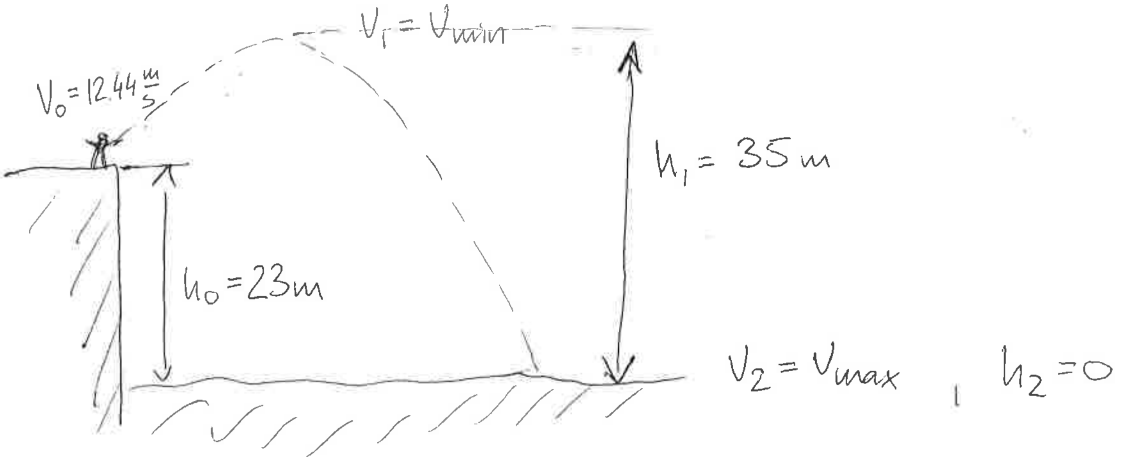
(c) $\boxed{F_c = \frac{m v^2}{r} = 2.42 * 10^5 \, N} = F_G$ gravitational force

3.) *Energy Conservation in Projectile Motion* *(14 pts.)*

A kid throws a stone with initial speed of 28mph from the edge of a cliff toward the open sea. The cliff is 23m above sea level. During flight, the stone reaches a maximal height of **7m** above the cliff. $(1m/s = 2.25mph)$

(a) Calculate the minimal speed of the stone during flight.

(b) Calculate the maximal speed of the stone during flight.



(a) minimal speed v_1 at top:

$$E_0 = E_1$$

$$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_1^2 + mgh_1$$

$$\Rightarrow \boxed{v_1} = \sqrt{v_0^2 + 2g(h_0 - h_1)} = \boxed{4.2\,\frac{m}{s}}$$

(b) maximal speed at bottom:

$$\boxed{v_2 = \sqrt{v_0^2 + 2g(h_0 - h_2)} = 24.6\,\frac{m}{s}}$$

4.) *Power and Uphill Acceleration* *(18 pts.)*

A sports car ($m = 1.3 tons$) accelerates from 0 to $60 mph$ in $6.8s$ on a horizontal road. ($1 m/s = 2.25 mph$, $1\, ton$=$1000\, kg$)

(a) How much net work is done on the car in the process?

(b) What average power is being applied to the car?

(c) In a different run (with the same car), the acceleration process is repeated on an uphill road. The car covers a vertical height difference of $25m$. How much longer does it take to go from 0-60?

(a) $$\boxed{W_{net}} = \Delta K = K_f - K_i = \frac{1}{2} m v_f^2 = \boxed{4.62 * 10^5 J}$$

(b) $$\boxed{\bar{P} = \frac{W}{\Delta t}} = 6.797 * 10^4 = \boxed{91.15 \text{ hp}}$$

(c) $$W_{net} = \Delta K + \Delta U = \frac{1}{2} m v_f^2 + mgh = 7.807 * 10^5 J$$

$$\Rightarrow \boxed{\Delta t = \frac{W_{net}}{\bar{P}} = 11.5 s}$$ i.e. 4.7s longer

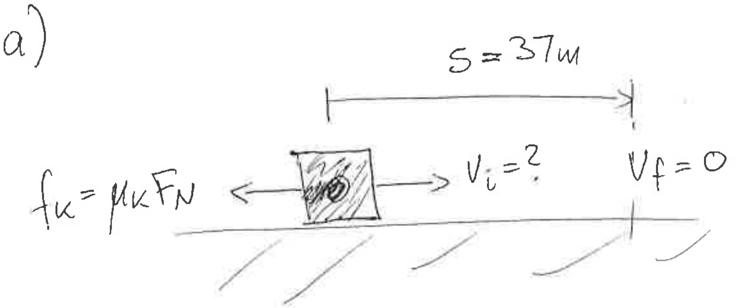
5.) *Ballistic Sliding Block* *(16 pts.)*

A cannon from the middle ages is shot at a giant wood block (mass 0.75 *tons*). The projectile, a spherical rock of mass 80 *kg*, hits the block in horizontal direction and gets stuck in it. The block recoils and slides on a horizontal gravel surface for 6.3m before coming to rest. The kinetic friction coefficient between block and gravel surface is 0.18.

(a) What was the speed of the block+rock right after the rock got stuck?

(b) What was the speed of the rock just before it hit the wood block?

(a)



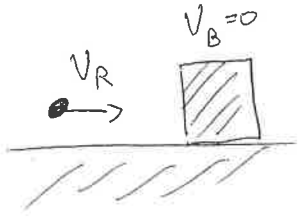
$$W_{nc} = \Delta K$$

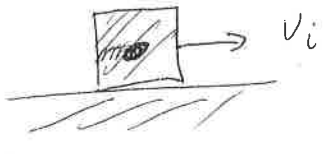
$$-f_k s = K_f - K_i = -K_i = -\frac{1}{2}(m_B + m_R) v_i^2$$

$$\Rightarrow \quad v_i^2 = \frac{2 f_k s}{(m_B + m_R)} = \frac{2\mu_k s (m_B + m_R) g}{(m_B + m_R)} = 2\mu_k s g$$

$$\boxed{v_i = \sqrt{2\mu_k g s}} = \boxed{4.71\ m/s}$$

(b)





$$m_R v_R = (m_B + m_R) v_i$$

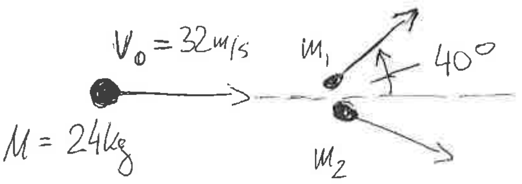
$$\Rightarrow \boxed{v_R = \frac{(m_B + m_R)}{m_R} v_i} = \boxed{48.9\ \frac{m}{s}}$$

6.) *Projectile Break-up in 2 Dimensions* *(16 pts.)*

A middle-aged stone shell ($m = 24\,kg$), filled with explosive powder (negligible mass), is traveling horizontally due East at a speed of $32\,m/s$ when it suddenly breaks up into two fragments of mass $m_1 = 10\,kg$ and $m_2 = 14\,kg$. Right after break-up both fragments are still traveling in the horizontal plane, with the lighter one heading 40° North of East with a speed of $28\,m/s$. Ignore gravity in this problem.

(a) What is the speed of the heavier fragment right after break-up?
(*Hint: you need to keep track of both x and y components*)

(b) Calculate the total kinetic energies before and after break-up and then decide whether the powder ignited ($W_{nc} > 0$) or not ($W_{nc} < 0$).



(a) $$P_o = P_f$$

$$V_{1x} = V_1 \cos 40^\circ$$

$$M V_{ox} = m_1 V_{1x} + m_2 V_{2x}$$

$$V_{ox} = V_o$$

$$\Rightarrow V_{2x} = \frac{1}{m_2}\left(M V_{ox} - m_1 V_{1x}\right) = 39.54\ m/s$$

$$0 = m_1 V_{1y} + m_2 V_{2y} \Rightarrow V_{2y} = -\frac{m_1}{m_2} V_{1y}, \quad V_{1y} = V_1 \sin 40^\circ$$

$$\Rightarrow V_{2y} = -12.86\ m/s \qquad \Rightarrow \boxed{V_2 = \sqrt{V_{2x}^2 + V_{2y}^2} = 41.57\ \frac{m}{s}}$$

(b) $$\boxed{W_{nc}} = \Delta K = K_f - K_i$$

$$= \left(\tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2\right) - \tfrac{1}{2} M v_o^2 = \boxed{3730\,J}$$

positive, i.e. the powder did explode!