
Partial Chiral Symmetry restoration and QCD Scalar Susceptibilities in Nuclear Matter

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- Chiral restoration in hadronic matter
- Scalar and pseudoscalar susceptibilities
- Susceptibility in a chiral effective theory
- Some consequences

A- CHIRAL RESTORATION IN HADRONIC MATTER

A1 -IN-MEDIUM SPECTRAL FUNCTIONS

Restructuring of the QCD vacuum (Dropping of the condensates)

Observable consequences in the excitation spectrum

In-medium hadronic spectral functions

$$\rho_h(\omega, \vec{q}) = -\frac{1}{\pi} \text{Im} D_h(\omega, \vec{q})$$

- Evolution of the “centroids” (masses) : no universal links between masses and condensates *i.e.* masses vs f_π
- Shape of the hadronic spectral functions and related quantities (χ)
- Softening, sharpening (σ) or broadening (K, ρ), dissolving of the resonances

A2- CHIRAL RESTORATION AND HADRON STRUCTURE

- Dropping of the quark condensate at finite ρ_B, T

$$\frac{\langle\langle\bar{q}q\rangle\rangle(\mu_B, T)}{\langle\bar{q}q\rangle_{vac}} = 1 - \sum_h \frac{\rho_s(\mu_B, T) \sigma_h}{f_\pi^2 m_\pi^2}$$

$$\begin{aligned}\sigma_h &= m_q \frac{\partial M_h}{m_q} = m_\pi^2 \frac{\partial M_h}{m_\pi^2} \\ \rho_s(\mu_B, T) &= \frac{\partial \Omega(\mu_B, T)}{\partial M_h}\end{aligned}$$

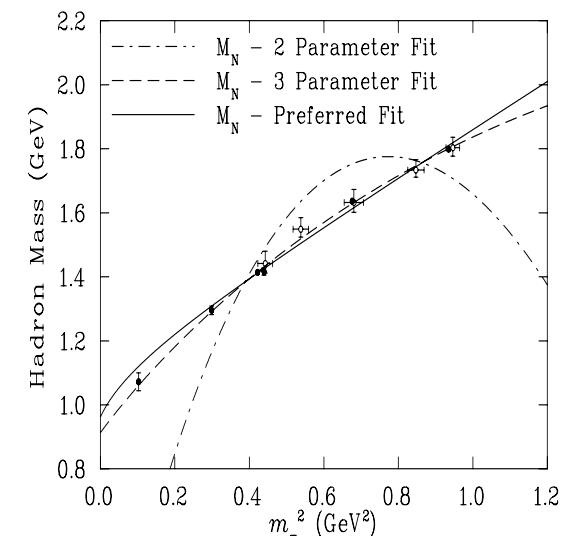
Dominated by lightest hadrons , heavy hadrons decouple from the condensate

- Nuclear matter, leading order

$$\frac{\langle\langle\bar{q}q\rangle\rangle(\rho)}{\langle\bar{q}q\rangle_{vac}} \simeq 1 - 0.35 \frac{\rho}{\rho_0} - \frac{T^2}{8 f_\pi^2}$$

- Extrapolation of lattice data (Leinweber,Thomas)

- $M_{N,\Delta} = \alpha + \beta m_\pi^2 + \text{Pion loop}(\Lambda)$
- The specific contribution of the pion cloud is very important and depends on a scale : the nucleon size (hidden in χPT)
- $\sigma_N \simeq 50 MeV$, half of it from the pion cloud

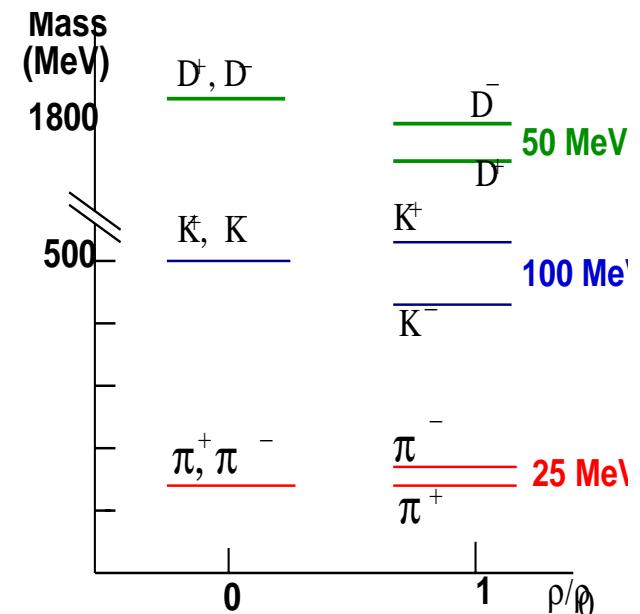


A3- IN-MEDIUM MASS SPLITTING

- Pattern of symmetry breaking generated by chiral dynamics

Deeply bound pionic states : In medium increase of the isovector scattering length b_1

$\Delta M_{\pi^-}^{exp} = 23 \sim 27 MeV$ in the center of Pb



- Chiral effective theory

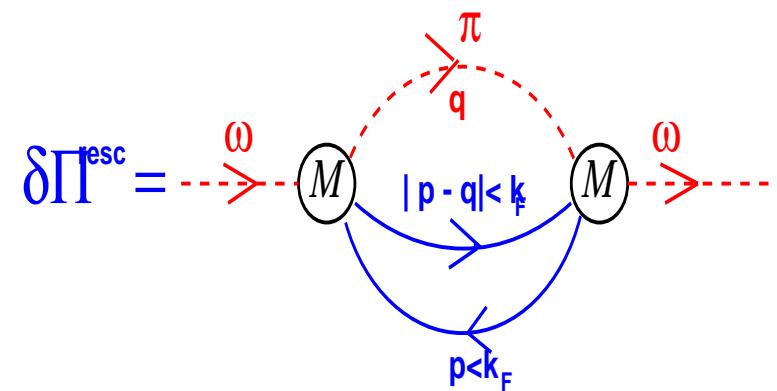
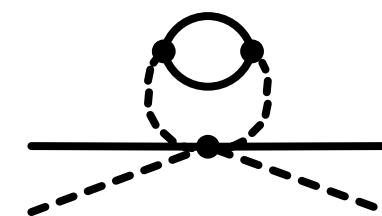
Vertex correction : pion cloud effect

$$\frac{b_1^*}{b_1} = 1 + \frac{\sigma_N \rho}{f_\pi^2 m_\pi^2} - \frac{7}{6} \frac{\Sigma_N^{(\pi)} \rho}{f_\pi^2 m_\pi^2} \simeq 1 + 0.18 \frac{\rho}{\rho_0}$$

Energy dependance of rescattering in presence of Pauli correlations

Everything together

$$\frac{b_1^*}{b_1} = 1.19 \quad \text{at} \quad \rho = 0.5\rho_0$$

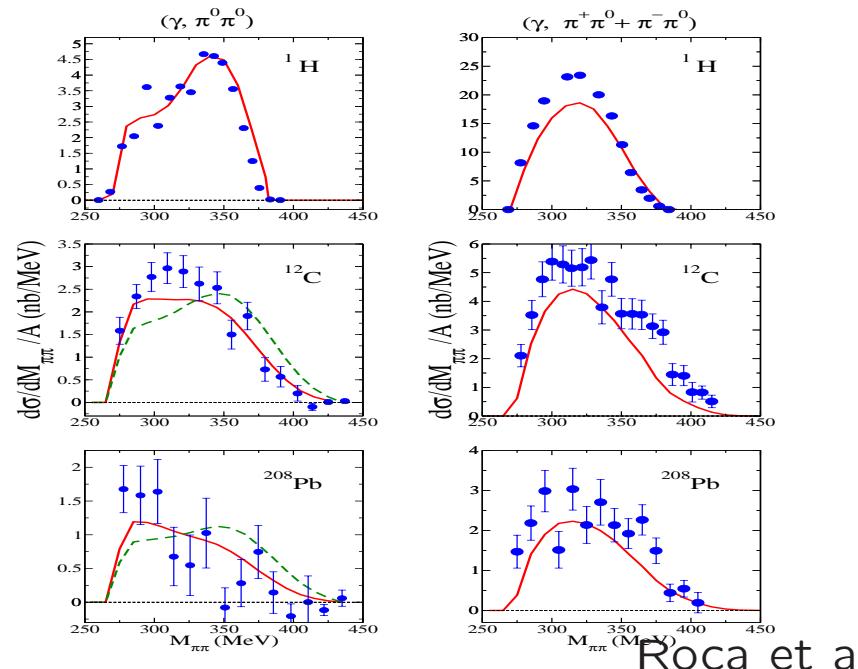


A4- 2π PROCESSES AND CHIRAL SYMMETRY

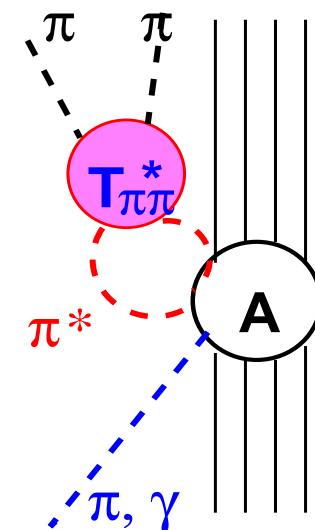
- $A(\pi, \pi\pi)$ (CHAOS, CB), $A(\gamma, \pi\pi)$ (TAPS)

Downwards shift of the $\pi\pi$ invariant mass distribution in the scalar-isoscalar channel $I = J = 0$

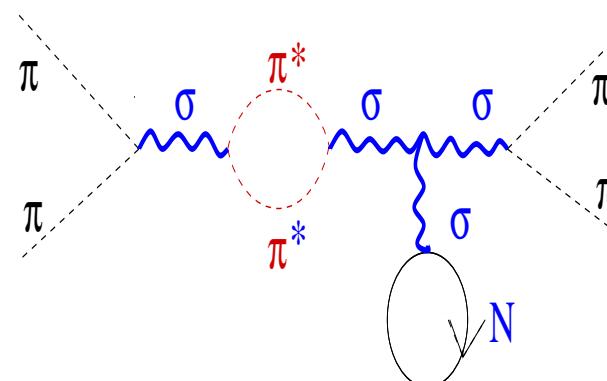
- What is the role of
 - Chiral Dynamics
 - Chiral restoration ?



Roca et al



- In-medium $I = J = 0$ $\pi\pi$ interaction
 - In-medium pions (Schuck-Chanfray)
 - Dropping of m_σ, f_π (Hatsuda-Kunihiro)



- **Dropping of the sigma mass**

- Associated with partial chiral restoration

Linear sigma model (Hatsuda) : 30% dropping at $\rho = \rho_0$

$$\frac{m_\sigma^{*2}}{m_\sigma^2} = 1 + 3 \frac{\langle s \rangle}{f_\pi} \quad \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle_{vac}} = 1 + \frac{\langle s \rangle}{f_\pi} - \frac{\langle \phi_\pi^2 \rangle}{2 f_\pi^2}$$

- But constraints from nuclear matter stability
- Closely related to the $p - h$ contribution to the nuclear scalar susceptibility

- **In-medium modified two-pion interaction**

- Closely related to the pion cloud contribution to the nuclear scalar susceptibility
- Also linked to chiral restoration

In-medium Scalar and Pseudoscalar susceptibilities

Partial Chiral symmetry restoration ?

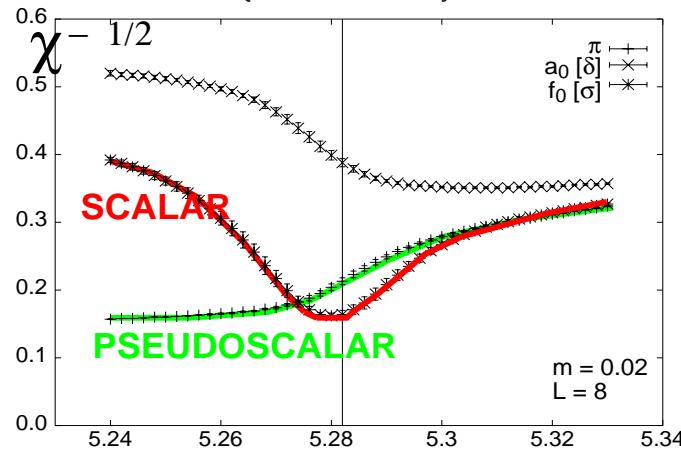
B- SCALAR AND PSEUDOSCALAR SUSCEPTIBILITIES

B1- FLUCTUATIONS OF THE CONDENSATE

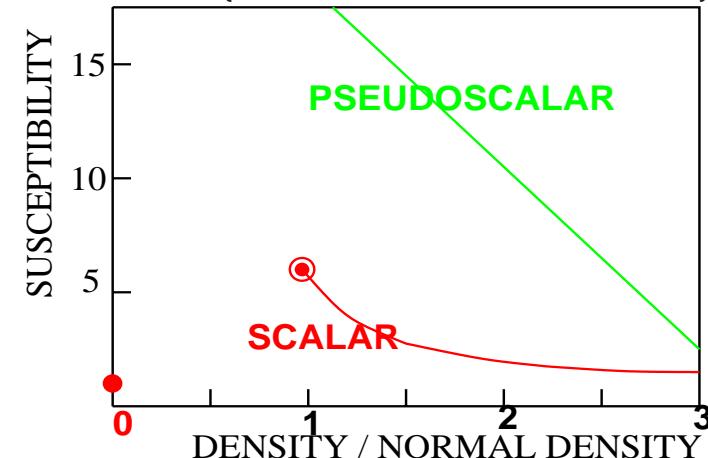
Scalar susceptibility : from the scalar correlator i.e. the correlator of the scalar quark density fluctuations

$$\chi_s = \frac{\partial \langle \langle \bar{q} q \rangle \rangle}{\partial m} = 2 \int dt d\vec{r} \langle \langle \delta \bar{q} q(0, 0), \delta \bar{q} q(\vec{r}, t) \rangle \rangle$$

Thermal susceptibility on Lattice
(Karsch)



Finite density : effective chiral theory (M. Ericson, G. C.)



Chiral Restoration : $\chi_S \rightarrow \chi_{PS}$

B2- SCALAR SUSCEPTIBILITY

$$\chi_S = \frac{\partial \langle \bar{q}q \rangle}{\partial m_q} = 2 \int dt' d\mathbf{r}' \Theta(t - t') \langle -i [\bar{q} q(0), \bar{q} q(\mathbf{r}', t')] \rangle$$

$$\chi_S = \left(\frac{\partial^2 \omega}{\partial m_q^2} \right)_\mu = Re G_S(\omega = 0, \vec{q} \rightarrow 0) = \int_0^\infty d\omega \left(-\frac{2}{\pi\omega} \right) Im G_S(\omega, \vec{q} = 0)$$

- A strong contribution of **low energy nuclear excitations** is expected
- Linear sigma model $\bar{q}q \rightarrow \frac{\langle \bar{q}q \rangle_{vac}}{f_\pi} \sigma$: The nuclear susceptibility is related to the **in-medium σ propagator**

B3- PSEUDOSCALAR SUSCEPTIBILITY

$$\chi_{PS} = 2 \int dt' d\mathbf{r}' \Theta(t - t') \langle -i [\bar{q} i\gamma_5 \frac{\tau_\alpha}{2} q(0), \bar{q} i\gamma_5 \frac{\tau_\alpha}{2} q(\mathbf{r}', t')] \rangle = \frac{\langle \bar{q}q \rangle(\rho)}{m_q}$$

(using $\partial^\mu \mathcal{A}_\mu^\alpha(x) = 2 m_q \bar{q} i\gamma_5 \frac{\tau_\alpha}{2} q(x)$ and soft pion theorem))

→ χ_{PS} behaves like the **chiral condensate**

B4- LEADING ORDER ESTIMATE

- Grand Potential : $\omega = \int \frac{4d^3p}{(2\pi)^3} (E_p - \mu) \theta(\mu - E_p)$
- Chiral condensate :

$$\langle \bar{q}q \rangle(\rho) - \langle \bar{q}q \rangle_{vac} = \frac{1}{2} \left(\frac{\partial \omega}{\partial m_q} \right)_\mu = \frac{1}{2} \frac{\partial M}{\partial m_q} \left(\frac{\partial \omega}{\partial M} \right)_\mu \equiv \frac{\sigma_N}{2m_q} \rho_S$$

σ_N : nucleon sigma term, ρ_S : scalar density

- Nuclear susceptibility :

$$\chi_S(\rho) = (\chi_S)_{vac} + \rho_S \frac{\partial}{\partial m_q} \left(\frac{\sigma_N}{2m_q} \right) + \frac{\sigma_N}{2m_q} \left(\frac{\partial \rho_S}{\partial m_q} \right)_\mu \equiv \rho_S \chi_S^N + \chi_S^{nuclear}$$

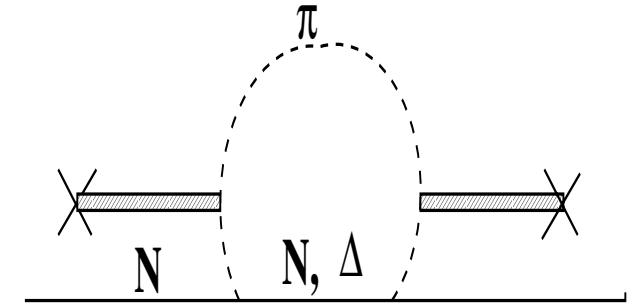
χ_S^N : nucleonic scalar susceptibility

- **Nucleonic contribution** : dominated by the pion cloud through the Leading non analytical contribution

$$\chi_S^N = -\frac{2\langle \bar{q}q \rangle_{vac}^2}{f_\pi^4 m_\pi} \frac{9}{64\pi} \left(\frac{g_A}{f_\pi}\right)^2 \left(\frac{\Lambda}{\Lambda+m_\pi}\right)^4$$

LNAC : $\rho_S \chi_S^N = 0.08 (\chi_{PS})_{vac}$

With $\Delta + FF + Pauli$: $\boxed{\rho_S \chi_S^N \simeq 0.04 - 0.05 (\chi_{PS})_{vac}}$



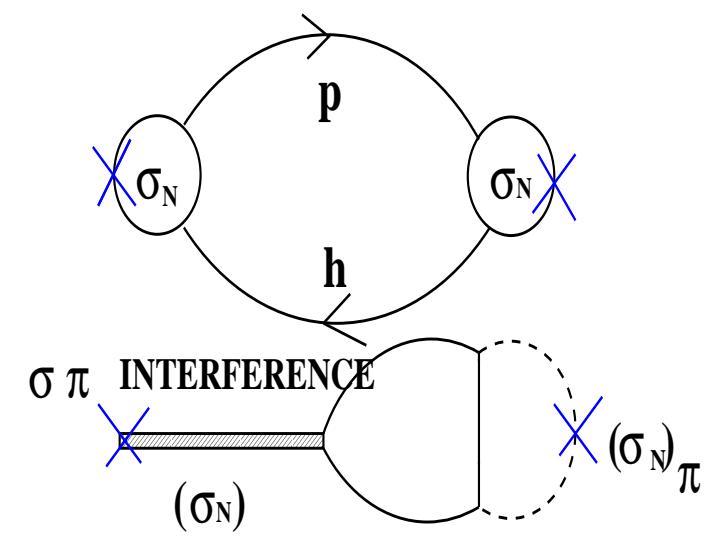
- **Nuclear (p-h) excitation contribution** : Compressibility K

$$\chi_S^{nuclear} = \frac{\sigma_N^2}{2m_q^2} \left(\frac{\partial \rho_S}{\partial M} \right)_\mu \equiv \frac{\sigma_N^2}{2m_q^2} \left(-\frac{2p_F M_N^2}{\pi^2} \right)$$

$$\left(-\frac{2p_F M_N^2}{\pi^2} \right) = \Pi_{ph}^0(\omega = 0, \vec{q} \rightarrow 0) = - \left(\frac{9\rho}{K} \right)_{\rho=\rho_0}$$

Sigma model : $\sigma_N = M_N \frac{m^2 \pi}{m_\sigma^2} + \sigma_N^{(\pi)}$

$\boxed{\chi_S^{nuclear} = 0.35 (\chi_{PS})_{vac}}$



C- SUSCEPTIBILITY IN A CHIRAL EFFECTIVE THEORY

C1- BASICS

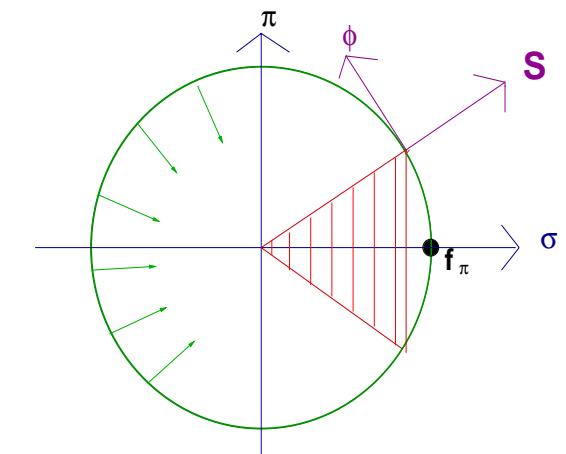
- Order parameter : $\mathcal{M} = \left(\frac{\bar{q}q}{2}\right) + i\vec{\tau} \cdot \left(i\bar{q}\gamma_5 \frac{\vec{\tau}}{2} q\right) = \boxed{\sigma + i\vec{\tau} \cdot \vec{\pi}}$
- From Cartesian to Polar coordinates

$$\sigma + i\vec{\tau} \cdot \vec{\pi} = (f_\pi + s) \exp \left[i \frac{\vec{\tau} \cdot \phi}{f_\pi} G \left(\frac{\phi^2}{f_\pi^2} \right) \right]$$

- Pion ϕ : phase fluctuation
- Scalar field $s = S - f_\pi$: amplitude fluctuation
- $\frac{\langle \bar{q}q \rangle_{medium}}{\langle \bar{q}q \rangle_{vacuum}} = 1 - \frac{\langle \phi_\pi^2(\rho) \rangle}{2f_\pi^2} - \frac{|\langle s(\rho) \rangle|}{f_\pi}$
- Effective Lagrangian

$$\begin{aligned} \mathcal{L} &= i\bar{N}\gamma^\mu\partial_\mu N - M_N^*(s)\bar{N}N + \frac{1}{2}\partial^\mu s\partial_\mu s - \frac{m_\sigma^2 - m_\pi^2}{8f_\pi^2} \left(s^2 + 2f_\pi s + \frac{2f_\pi^2 m_\pi^2}{m_\sigma^2 - m_\pi^2} \right)^2 \\ &\quad + \mathcal{L}_\omega + \mathcal{L}_{\pi NN}^{pwave} + \mathcal{L}_\pi + \mathcal{L}_{WT} \end{aligned}$$

$$M_N^*(s) = M_N \left(1 + \frac{s}{f_\pi} \right) \quad \text{BUT NO MATTER STABILITY}$$



C2- MATTER STABILITY IN CHIRAL THEORIES

“Shifted” vacuum with chiral order parameter

$$\bar{S} < f_\pi$$

Energy density : $\epsilon(\rho, \bar{S}) = \sum_{p < p_F} \sqrt{p^2 + M_N^*(\bar{S})} + V(\bar{S}) + C_V \rho^2$

- NJL (Bentz, Thomas, Birse) : Nucleon $\equiv qq - q$ state. $M_q^* = g_q \bar{S}$
- Chiral QHD : $S = f_\pi + s \equiv$ chiral invariant scalar field : $M_N^* = g_{sNN} \bar{S}$

$$g_{sNN}^*(\bar{S}) = \frac{\partial M_N^*}{\partial \bar{S}} \text{ drops}$$

Needed to stabilize nuclear matter

- NJL : Infrared cutoff : simulate confinement
- QMC : Polarization of the quark WF : nucleon structure

Nuclear saturation vs Nucleon structure and QCD (lattice)

C3- MEAN FIELD : IN-MEDIUM SIGMA MASS AND NUCLEAR SCALAR SUSCEPTIBILITY

- Include the scalar response κ_{NS} of the nucleon to a scalar field

$$M_N^* = M_N \left(1 + \frac{\bar{s}}{f_\pi} \right) + \frac{1}{2} \kappa_{NS} \bar{s}^2$$

- Minimization : $\frac{\partial \varepsilon}{\partial \bar{s}} = g_S^* \rho_S + V'(\bar{s}) = 0$, $g_S^*(\bar{s}) = \frac{\partial M_N^*}{\partial \bar{s}}$ drops with ρ
- In-medium sigma mass : $m_\sigma^{*2} = \frac{\partial^2 \varepsilon}{\partial \bar{s}^2} = V''(\bar{s}) + \kappa_{NS} \rho_S$
- χ_S related to in-medium sigma propagator dressed by ph excitations

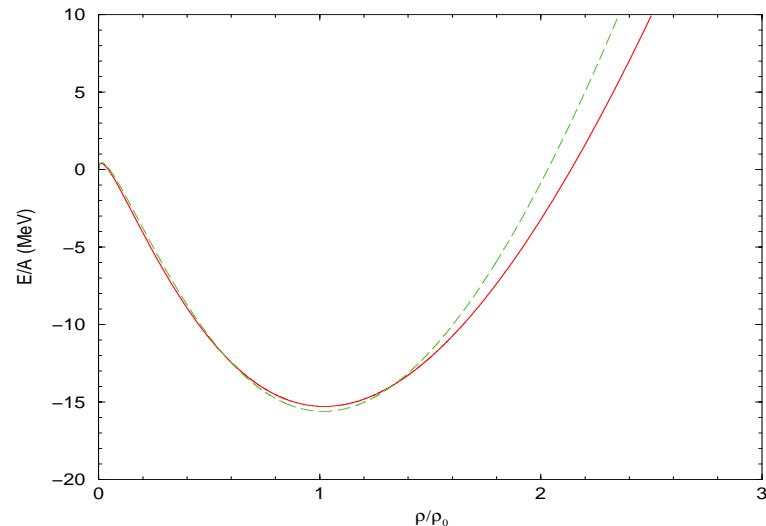
$$\chi_S = -2 \frac{\langle \bar{q}q \rangle_{vac}^2}{f_\pi^2} \left(-\frac{1}{m_\sigma^{*2}} + \frac{1}{m_\sigma^{*2}} \Pi_{SS}(0) \frac{1}{m_\sigma^{*2}} \right)$$

$\Pi_{SS}(0)$ is the full scalar polarization propagator (related to K)

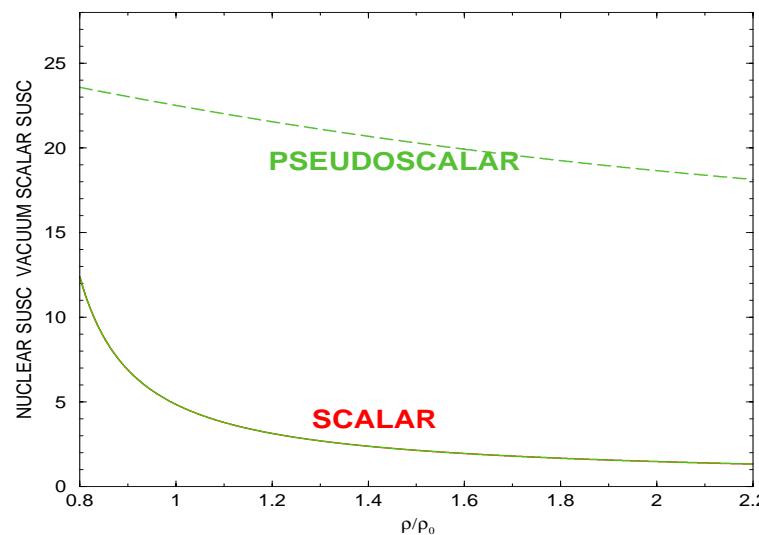
$$\Pi_{SS}(0) = g_S^{*2} \frac{M_N^*}{E_F^*} \Pi_0(0) \left[1 - \left(\frac{g_\omega^2}{m_\omega^2} \frac{E_F^*}{M_N^*} - \frac{g_S^{*2}}{m_\sigma^{*2}} \frac{M_N^*}{E_F^*} \right) \Pi_0(0) \right]^{-1}$$

- Parameters : m_σ ($\pi\pi$ phase shifts), g_ω and κ_{NS}

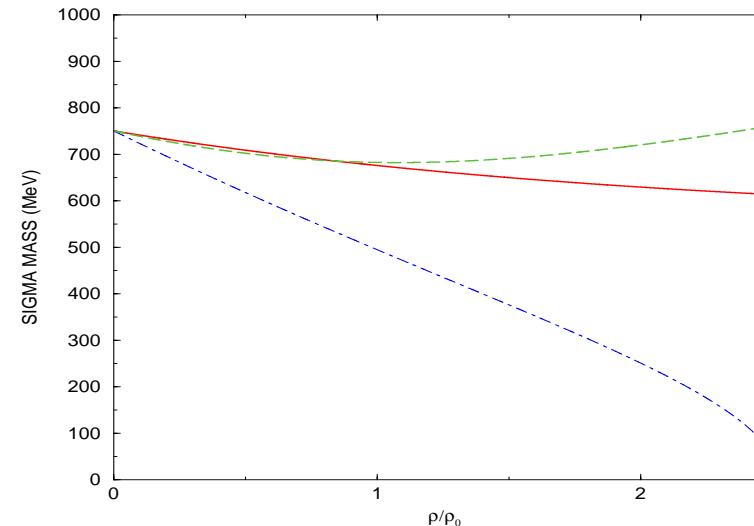
- 2 sets of parameters



- Nuclear susceptibility



- In-medium sigma mass

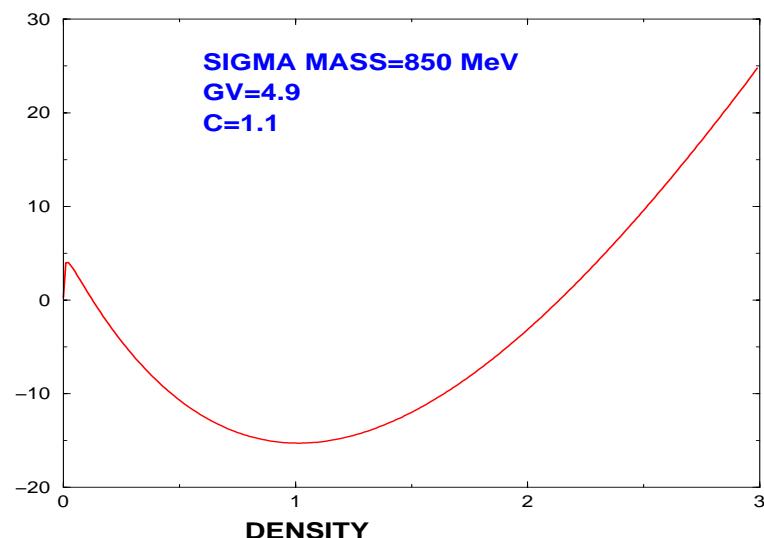


- Nucleon structure effect compensates the chiral dropping !
- κ_{NS} Probably too large
- → Introduce pion loops (CHIPT or pionic correlation energy)

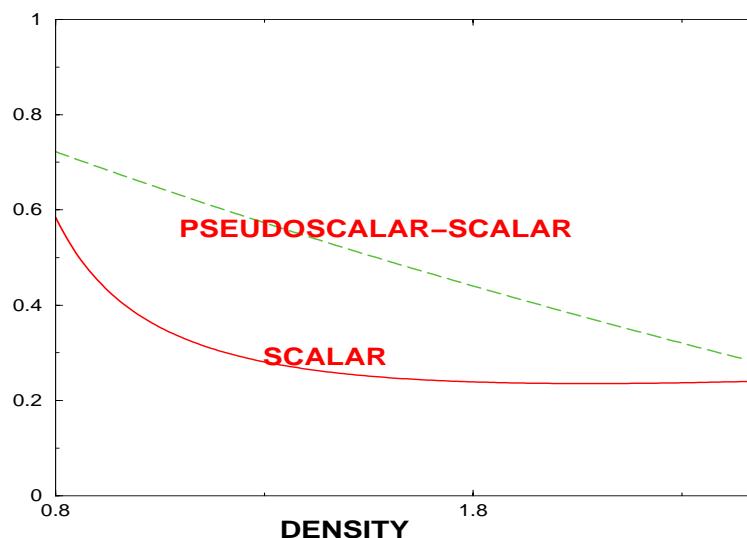
- With pionic contribution to the nucleon sigma term and pion Fock term

$$\chi_S = -2 \frac{\langle \bar{q}q \rangle_{vac}^2}{f_\pi^2} \left(-\frac{1}{m_\sigma^{*2}} + \frac{1}{m_\sigma^{*2}} \left(\frac{\sigma_N^{(\pi)} + \sigma_N^{(\sigma)}}{\sigma_N^{(\sigma)}} \right)_{eff}^2 \Pi_{SS}(0) \frac{1}{m_\sigma^{*2}} \right)$$

BINDING ENERGY



$CHI/CHIPS_{vac}$



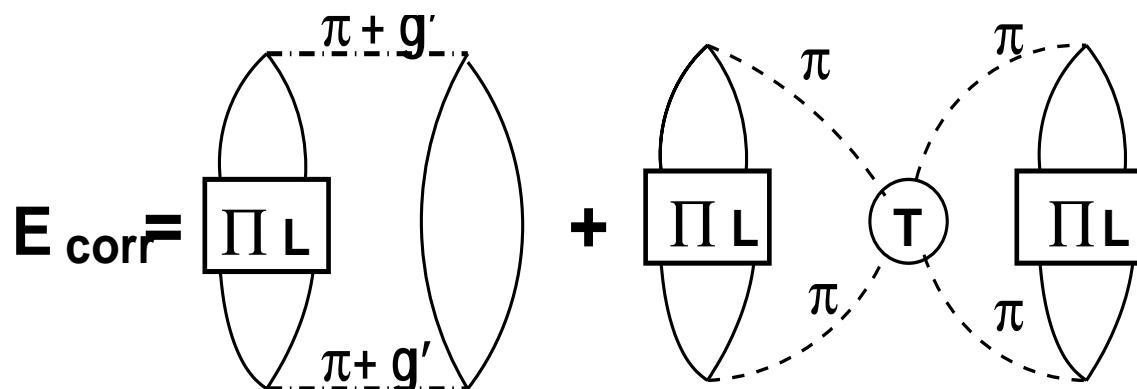
C4- PION CLOUD CONTRIBUTION TO THE IN-MEDIUM SUSCEPTIBILITY : i.e., pion loop or in-medium modified pion cloud contribution to the nucleonic susceptibility

- **Correlation energy** (to be incorporated numerically in the EOS)

$$E_{corr+Fock} = \int_0^1 \frac{d\lambda}{\lambda} \left(\int \frac{id^4 q}{(2\pi)^4} \lambda^2 D_{0\pi}(q) \Pi_L(q; \lambda) + \lambda^2 \frac{m_\pi^2}{12 \bar{S}^2} \langle \phi^2 \rangle^2 \right)$$

$\Pi_L(q; \lambda) = \Pi_0(q) / (1 - \lambda^2 (g' + V_{\pi exch}) \Pi_0(q))$ is the full longitudinal spin-isospin polarization propagator and $\Pi_0(q)$ is the Lindhardt function including (a slightly in-medium modified) $g_{\pi NN}$

$\langle \phi^2 \rangle = 3 \int \frac{id^4 q}{(2\pi)^4} \vec{q}^2 D_{0\pi}^2(q) \Pi_L(q)$ is the in-medium modified pion scalar density.



- **In-medium quark condensate** (Feynman-Hellman theorem)

$$\langle \bar{q}q \rangle_{Medium}^{\pi cloud} = \frac{m_\pi^2}{2 m_q} \frac{\partial E_{corr+Fock}}{\partial m_\pi^2} = -\langle \bar{q}q \rangle_{Vac} \frac{\bar{S}}{f_\pi} \frac{\langle \phi^2 \rangle}{2 \bar{S}^2} \left(1 + \frac{\langle \phi^2 \rangle}{12 \bar{S}^2} \right) \equiv \frac{1}{2m_q} \langle H_{\chi SB} \rangle_{Medium}^{\pi cloud}$$

Compatible with χPT at finite T (Gasser-Leutwyler, GC-Ericson-Wambach)

- **Susceptibility from the pion cloud**

$$\chi_S^{\pi cloud} = \frac{\partial \langle \bar{q}q \rangle_{Medium}^{\pi cloud}}{\partial m_\pi^2} = -2 \frac{\langle \bar{q}q \rangle_{Vac}^2}{f_\pi^2} \frac{\bar{S}}{f_\pi} \frac{3}{2 \bar{S}^2} \left(\frac{G(0) - G_{vac}(0)}{1 - \frac{V_{\pi\pi}}{2} G(0)} \right)$$

$G(E) = \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{i dq_0}{2\pi} D_\pi(\mathbf{q}, q_0) D_\pi(-\mathbf{q}, E - q_0)$ is the in-medium two-pion propagator
(pion dressed by $p-h$ and $\Delta-h$)

RESULTS

- Vacuum : $(\chi_S)_{vac} = 0.04$ $(\chi_{PS})_{vac}$
- First order (one p-h insertion) : $\delta \chi_S^{nuclear} = \rho_S \chi_S^N = 0.064$ $(\chi_{PS})_{vac}$
- Full calculation : (full pion dressing + $\pi\pi$ rescatt.) $\delta \chi_S^{nuclear} = 0.10$ $(\chi_{PS})_{vac}$

D- SOME CONSEQUENCES

D1- THE SCALAR SUSCEPTIBILITY AND $\pi\pi$ PRODUCTION

- The $\pi\pi$ T matrix

$$T_{\pi\pi}(E) = V_{\pi\pi}(E) + V_{\pi\pi}(E) G(E) T_{\pi\pi}(E)$$

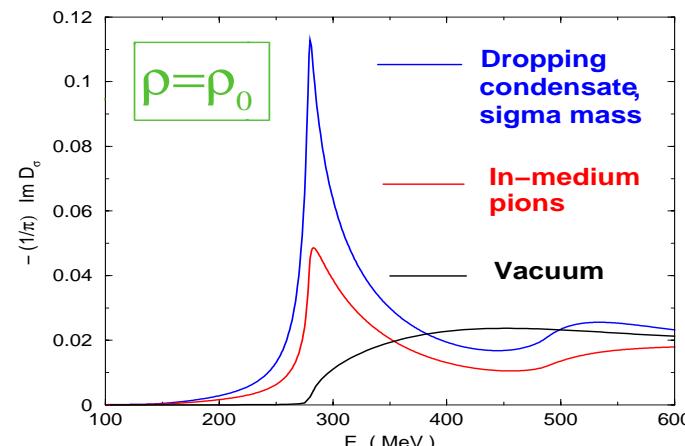
$$G(E) = \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{i dq_0}{2\pi} D_\pi(\mathbf{q}, q_0) D_\pi(-\mathbf{q}, E - q_0)$$

- $T_{\pi\pi}(E)$ calculable in unitarized χ PT (Valencia) or in sigma model (Lyon-Darmstadt) but the medium effects near the 2π threshold are actually independant of the sigma mass if the sigma mass remains stable.
- The medium effects seen in $\pi\pi$ production experiment are embedded in $G(E \simeq 2m_\pi)$, the in-medium two-pion propagator (pion dressed by $p-h$ and $\Delta-h$)
- The in-medium modification of the pionic susceptibility is embedded in $G(E=0)$

Sigma model (phase shifts)

$$T_{\pi\pi}(E) = \frac{6\lambda(E^2 - m_\pi^2)}{1 - 3\lambda G(E)} D_\sigma(E)$$

$$D_\sigma(E) = (E^2 - m_\sigma^2 - \frac{6\lambda^2 f_\pi^2 G(E)}{1 - 3\lambda G(E)})^{-1}$$



- **The full nuclear scalar susceptibility**

- The σ (Chiral partner of the π) vs the s (Chiral invariant)
- Strong $\sigma\pi\pi$ coupling weak derivative $s\pi\pi$ coupling
- Express the pion-pion T matrix in terms of either the σ propagator or the s propagator

$$\chi_S = -2 \frac{\langle \bar{q}q \rangle_{vac}^2}{f_\pi^2} D_\sigma(0)$$

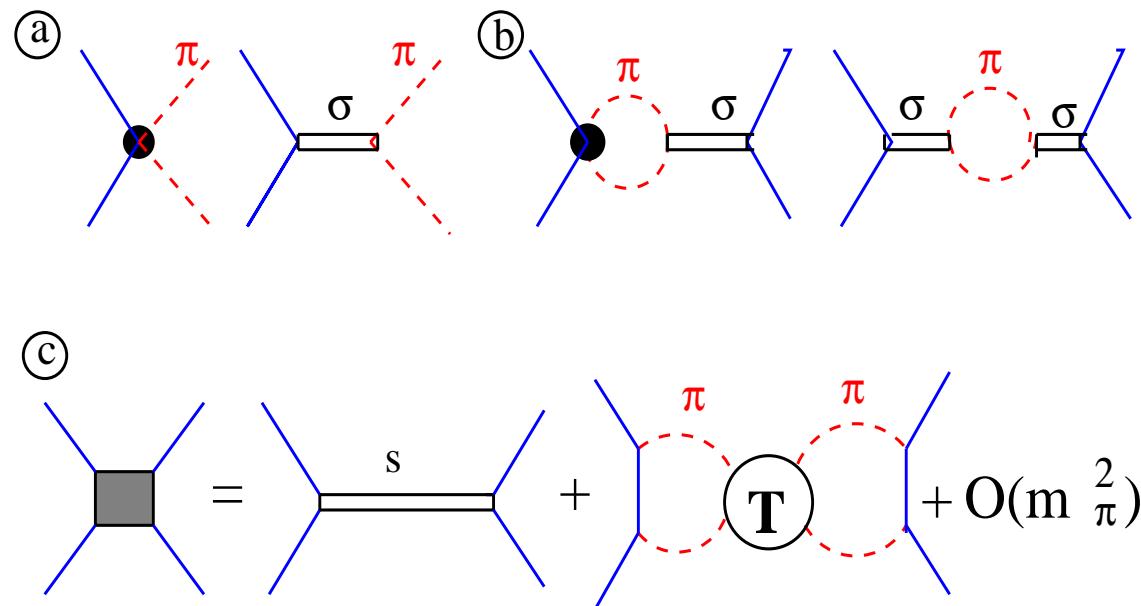
$$D_\sigma(E) = D_s(E) + \frac{3}{2S_\pi^2} \left(1 - 2 \frac{E^2 - m_\pi^2}{E^2 - m_\sigma^2} \right) \tilde{G}$$

\tilde{G} is the full two-pion propagator : $G = G + \frac{1}{2} G V_{\pi\pi} \tilde{G}$

- The chiral invariant s mode is the exchanged meson in the scalar NN potential and is free of pionic many-body effect : weak coupling to in-medium modified 2π states. It couples to (p-h) low energy nuclear excitations and gives the nuclear piece of the susceptibility. This is the sigma meson of nuclear physics.
- The medium effects in the σ propagator and in two-pion processes come from the second 2π modes. It gives the pion cloud piece of the scalar susceptibility.

D2- NUCLEAR PHYSICS IMPLICATIONS : THE NN SCALAR POTENTIAL

Nucleons exchange not only σ but also 2π states with delicate compensations (M. Birse)

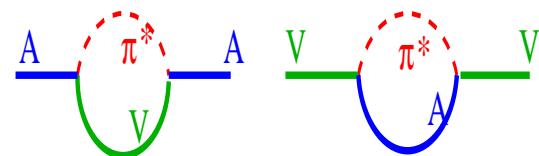


- THE NONLINEAR σ FORMULATION IS MUCH MORE ECONOMICAL
- THE MEDIUM EFFECT IN THE 2π PROCESSES ARE MOSTLY ABSENT IN THE NN SCALAR EXCHANGE

D3- CORRELATOR MIXING

- **Axial-vector mixing**

At finite temperature



$$\Pi_V^{\mu\nu}(q; T) = (1 - \epsilon) \Pi_V^{\mu\nu}(q; T = 0) + \epsilon \Pi_A^{\mu\nu}(q; T = 0)$$

$$\Pi_A^{\mu\nu}(q; T) = (1 - \epsilon) \Pi_A^{\mu\nu}(q; T = 0) + \epsilon \Pi_V^{\mu\nu}(q; T = 0)$$

$$\epsilon = \frac{T^2}{6 f_\pi^2} = \frac{2}{f_\pi^2} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{n(\omega_k)}{\omega_k} = \frac{2}{3} \frac{<<\Phi^2>>}{f_\pi^2}$$

Also at finite density (G.C , M. Ericson), (Rapp-Wambach), (Wolf) : **Dilepton production**

- **Scalar-pseudoscalar mixing**

$$D_\sigma(E) = D_s(E) + \frac{3}{2S_\pi^2} \left(1 - 2 \frac{E^2 - m_\pi^2}{E^2 - m_\sigma^2} \right) G \quad D_\sigma(E, T = 0) \simeq D_s(E) \simeq 1/E^2 - m_\sigma^2$$

For soft thermal pions, $q \ll E$:

$$3G(E, T) = 3 \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{i dq_0}{2\pi} D_\pi(\mathbf{q}, q_0) D_\pi(-\mathbf{q}, E - q_0) \simeq \frac{<<\Phi^2>>}{E^2 - m_\pi^2}$$

$$D_\sigma(E, T) = \left(1 - \frac{<<\Phi^2>>}{S_\pi^2} \right) D_\sigma(E, T = 0) + \frac{<<\Phi^2>>}{2S_\pi^2} D_\pi(E, T = 0)$$

CONCLUSION

- Sizeable effects in the nuclear scalar susceptibility
- Scalar and pseudoscalar susceptibilities becomes closer → partial chiral restoration
- Nucleon structure effect compensates the chiral dropping of the sigma mass ; related to the p-h piece of the scalar susceptibility
- $\sigma - 2\pi$ medium effects in two-pion processes related to the pionic nuclear scalar susceptibility. Linked to a scalar-pseudoscalar mixing effect associated with partial chiral restoration.
- The chiral invariant s mode couples to (p-h) low energy nuclear excitations and gives the nuclear piece of the susceptibility.
- Extension at higher density and/or temperature. How to constrain the effective theories ? Renormalization group ?