Name: PHY401 (Spring 2013), 02/26/13

Homework Assignment #5

(*Due Date*: Tuesday, March 05, 05:30 pm, in class)

5.1 *Poisson Equation for Electric Dipole* (cf. Ex. 5.7) *(2+1+1+2 pts.)*
Write a FORTRAN program to numerically calculate the potential of an electric dipole, i.e., two point charges $Q/\epsilon_0 = \pm 1$ separated by a distance $a = 0.5$ around the origin. Solve the 3D Poisson equation in Cartesian coordinates but impose spherical boundary conditions, $V(R)$=0, at a large distance R=10 or so.

(a) Starting from an initial condition of zero potential use the Jacobi relaxation algorithm with appropriate numerical tolerance and grid density to obtain the (converged) solution. Plot the equipotential lines, as well as $V(r)$ for x=y=z.

(b) Compute the magnitude of the electric field, $|\vec{E}(r)|$ (r: distance from dipole center), extract the exponent n for the large-distance behavior, $E(r) \propto r^{-n}$. Do you find the theoretically expected result for the dipole?

(c) Investigate how the number of required iteration steps, N_{iter}, increases with reducing the tolerance (error) limit, "*tol*", and plot $N_{iter}(tol)$.

(d) Write a second program by modifying the algorithm to using the Simultaneous Over Relaxation (SOR) method, and allow for variable grid density in different relaxation runs. Focus on the x-y plane only for which the optimal value of α is close to 2. For fixed accuracy in the solution (not total tolerance!), investigate (and plot) how the number of iteration steps, N_{iter}, depends on the number n=n_x=n_y of grid points, showing that for the Jacobi and SOR method, $N_{iter} \propto n^2$ and $N_{iter} \propto n$, respectively.

5.2 *Helmholtz Coils* (cf. Ex. 5.15) *(4 pts.)*
Use Biot-Savart's Law for the magnetic field induced by a thin-wire current,

$$d\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \, \frac{d\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \; , \tag{1}$$

to construct a FORTRAN code calculating the $\vec{B}$-field for an arrangement of 2 current loops (circular wires), each of radius r oriented parallel to the x-y plane and separated by a distance r (cf. also Fig. 5.18 in the textbook). Choose, e.g., $r = 0.5$, $\mu_0 I$=10 in SI units running counter-clockwise. Calculate and plot the x-, y- and z-components of the magnetic field for
(a) $x = y = 0$ as a function of z (check with the analytic result!)
(b) $y = z = 0$ as a function of x.