Name: PHY401 (Spring 2013), 02/05/13

Homework Assignment #3

(*Due Date*: Tuesday, February 12, 05:30 pm, in class)

3.1 *Driven Harmonic Motion with Damping* *(2+1+2+1+2+2 pts.)*

The motion of the physical pendulum is described by the differential equation

$$\frac{d^2\theta}{dt^2} = -\Omega^2 \sin(\theta) - 2\gamma\frac{d\theta}{dt} + \alpha_D \sin(\Omega_D t) \ . \tag{1}$$

where $\Omega^2 = g/l$. In the following, we will investigate this system at varying levels of approximation (use g=9.8 m/s^2, l=9.8 m, γ=0.25 s^{-1} unless otherwise specified).

(a) First consider simple harmonic motion (SHM): $\sin(\theta) \to \theta$, γ=0, α_D=0. Write a fortran code to compute $\theta(t)$; use both Euler and Euler-Cromer method, plot and compare your results (choose θ_0 as not to violate the linear approximation by more than 2%). For both methods, calculate analytically the total energy, E_{i+1}, in terms of E_i. In particular, show that
$E_{i+1} = E_i + (KE_i - PE_i)\Omega^2(\Delta t)^2 - \frac{1}{2}ml^2\Omega^4\theta_i(\omega_i + \omega_{i+1})(\Delta t)^3$
for the Euler-Cromer method and explain why this is an improvement.

(b) Relax the linear approximation and re-calculate and -plot your results for $\theta(t)$. Use θ_0 as in part (a), as well as $\pi/2$ and close to π. Explain the differences to SHM.

(c) Additionally include damping and a moderate driving force, $\alpha_D = 0.4 rad/s^2$, in your FORTRAN code to numerically calculate $\theta(t)$ using the Euler-Cromer method. Plot $\theta(t)$ and $\omega(t) = d\theta/dt$, together with the external driving acceleration, $\alpha_D(t)$, over a sufficiently long time to reach the steady-state solution. Then extract the amplitude $\Theta_0(\Omega_D)$ and phase shift $\phi(\Omega_D)$ for at least 10 different driving frequencies mapping out the resonance structure, and plot $\Theta_0(\Omega_D)^2$ and $\phi(\Omega_D)$. Compare these results to the analytical curves for the linear driven HM and comment on the difference, if any.

(d) For a driving frequency close to resonance, compute potential, kinetic and total energies, and plot them in the same graph over ca. 10 periods.

(e) For α_D=0.4 rad/s^2 and 1.2 rad/s^2, compute $|\Delta\theta(t)|$ for several pairs of trajectories with slightly different initial angle ($\Delta\theta_{\rm in}$ = 0.001 rad or so). Plot the results and estimate the (largest) Lyapunov exponent λ for each α_D.

(f) Generate the bifurcation diagram of the pendulum by taking snapshots of the amplitude (in the steady-state regime) at integer multiples of the driving period, and plotting all values for $\theta(nT_D)$ for each α_D, as a function of α_D. Start with α_D=1.4 rad/s^2 and move forward in small steps (e.g., 0.001 to begin with) to track the period doublings for at least 4 times. Estimate the Feigenbaum number.