

Homework Assignment #7

(Due Date: Wed, 12/05, 04:10 pm, in class; Show all your work for full/partial credit)

7.1 Nuclear Sizes, Binding and Shell Structure (1+2+2+1 pts.)

Consider the elements ${}^7_3\text{Li}$ (7.016), ${}^{40}_{20}\text{Ca}$ (39.963), ${}^{56}_{26}\text{Fe}$ (55.935), ${}^{62}_{28}\text{Ni}$ (61.928) and ${}^{197}_{79}\text{Au}$ (196.967) with their measured atomic masses in units of $u=931.5 \text{ MeV}/c^2$ in parentheses (use $m_p=938.3 \text{ MeV}/c^2$, $m_n=939.6 \text{ MeV}/c^2$ and $m_e=0.5 \text{ MeV}/c^2$).

- (a) Calculate their nuclear radii.
- (b) Calculate their nuclear binding energies per nucleon, E_B/A , from the above data.
- (c) Calculate the total nuclear mass per nucleon *including* the nucleon rest masses (but not the electrons) for the Fe-56 and Ni-62 nuclei (use improved accuracy, $u=931.494 \text{ MeV}/c^2$ and $m_e=0.511 \text{ MeV}/c^2$). Is the hierarchy the same as in part (b)?
- (d) Calculate the binding energy predicted by the liquid-drop model (see equation (1) in problem 7.2 below) for Li-7, Fe-56 and Au-197, and quantify the percentage deviation from the measured values obtained in part (b).
- (e) Classify all nuclei above according to the nuclear shell model as either non-magic, magic (in N or Z) or doubly magic (in N and Z).

7.2 Liquid Drop Model of Nuclei (2 pts.)

The empirical Weizsäcker formula for the binding energy of nuclei is given by

$$E_B = C_1 A + C_2 A^{2/3} + C_3 \frac{Z(Z-1)}{A^{1/3}} + C_4 \frac{(A-2Z)^2}{A} \quad (1)$$

with A : nuclear mass number, Z : nuclear charge in units of e , $C_1=-15.75 \text{ MeV}$, $C_2=17.8 \text{ MeV}$, $C_3=0.71 \text{ MeV}$ and $C_4=23.7 \text{ MeV}$.

- (a) Derive the value Z^* for the charge which minimizes the binding energy for a given A . Sketch the resulting “valley of stability”, $Z^*(A)$, in a plot.

7.3 Radioactivity (2 pts.)

In a nuclear reactor, several radioactive elements are produced, including Pu-239 (with a half-life of 24110 years) and Cs-137 (with a half-life of 30.17 years).

- (a) What is the predominant kind of radiation that the decays of these nuclei emit?
(*hint: consult the internet*)
- (b) What is the activity (in Ci) of 10 kg of (i) Pu-239, (ii) Cs-137?
(Assume a molar weight of 239 g and 137 g , respectively; $N_A=6 \cdot 10^{23}$)