Name(s): PHY-222 (Fall 2017), 11/30/17

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# Homework Assignment #7

(*Due Date*: Tue, 12/05, 05:30 pm, in class; Show all your work for full/partial credit)

7.1 *Nuclear Sizes, Binding and Shell Structure* *(1+2+2+1 pts.)*
Consider the elements $^{4}_{2}$He(4.0026), $^{16}_{8}$O(15.995), $^{56}_{26}$Fe(55.935), $^{62}_{28}$Ni(61.928) and $^{208}_{82}$Pb(207.977) with their experimentally measured masses in units of $u$=931.5 $MeV/c^2$ in parentheses (use $m_p$=938.3 $MeV/c^2$, $m_n$=939.6 $MeV/c^2$ and $m_e$=0.5 $MeV/c^2$).

(a) Caclulate their nuclear radii.

(b) Calculate their nuclear binding energies per nucleon, $E_B/A$.

(c) Calculate the total nuclear mass per nucleon *including* the nucleon rest masses (but not the electrons) for the Fe-56 and Ni-62 nuclei (use improved accuracy, $u$=931.494 $MeV/c^2$ and $m_e$=0.511 $MeV/c^2$). Is the hierarchy the same as in part (b)?

(d) Calculate the binding energy per nucleon predicted by the liquid-drop model (see equation (1) in problem 7.2 below) for all nuclei above, and quantify the percentage deviation from the measured values obtained in part (b).

(e) Classify all nuclei above according to the nuclear shell model as either non-magic, magic (in $N$ or $Z$) or doubly magic (in $N$ and $Z$).

7.2 *Liquid Drop Model of Nuclei and Nuclear Fission* *(4 pts.)*
The empirical Weizsäcker formula for the binding energy of nuclei is given by

$$E_B = C_1 A + C_2 A^{2/3} + C_3 \frac{Z(Z-1)}{A^{1/3}} + C_4 \frac{(A-2Z)^2}{A} \tag{1}$$

with $A$: nuclear mass number, $Z$: nuclear charge in units of $e$, $C_1$=−15.75 MeV, $C_2$=17.8 MeV, $C_3$=0.71 MeV and $C_4$=23.7 MeV.

(a) Derive the value $Z^*$ for the charge which minimizes the binding energy for fixed $A$. Sketch the resulting “valley of stability”, $Z^*(A)$, in a plot.

(b) How much fission energy is released in $^{235}_{92}U \rightarrow\ ^{144}_{56}Ba +\ ^{89}_{36}Kr + 2n$? How many kg of $^{235}U$ are spent in a nuclear reactor operating for 1 year at an output of 1000 MW (neglect any losses; 1 eV=1.6·10$^{-19}$J)?
($m(^{235}U) = 235.04u$ , $m(^{144}Ba) = 143.92u$ , $m(^{89}Kr) = 88.92u$)