

Homework Assignment #1

(Due Date: Thu, 09/14, 05:30 pm, in class; Show all your work for full/partial credit)

1.1 Momentum Conservation in Moving Frames (cf. Prob. 1.2.2 in textbook) (2 pts.)

Car-1 of mass $m_1 = 1300\text{kg}$, initially moving at a speed of 50 mph , collides head on with car-2 ($m_2 = 1000\text{kg}$) which is initially at rest. After the collision the 2 cars stick together.

- (a) Calculate the momentum and speed of the two cars right after the collision.
- (b) Transform initial and final momenta of both cars into a reference frame moving at 25 mph in the direction of the initial velocity of car-1 and check momentum conservation in this moving frame.

1.2 Time Dilation (cf. Prob. 1.5.10+13 in textbook) (2 pts.)

The positively charged pion (π^+) is an unstable particle with an “average” lifetime of $\tau_\pi = 2.6 \cdot 10^{-8}\text{s}$, *i.e.*, its abundance decreases as $N(t) = N_0 \exp(-t/\tau_\pi)$ after initial production. Suppose a π^+ is produced in the atmosphere, 12 km above the Earth surface, with a speed of $0.99998c$ heading straight down.

- (a) How far on average will the pion travel in Earth’s rest frame before decaying?
- (b) What is the probability the pion to make it to the Earth surface?
(or: out of 1 million such initial pions, how many will hit the Earth surface?)

1.3 Doppler Effect (cf. Prob. 1.5.16 in textbook) (2 pts.)

A professor argues to a judge that he was running a red light (wavelength $\lambda=650\text{nm}$) because it appeared orange (wavelength $\lambda=610\text{nm}$) to him on approach. The red-light fine is \$200, while speeding fines are \$20 for each 10 mph above the speed limit of 60 mph . What kind of speeding fine is the professor looking at?

1.4 Lorentz Transformation (cf. Probs. 1.6.19+21 in textbook) (4 pts.)

An observer standing on Earth observes spaceship travel.

- (a) First, 2 spaceships are heading in the same direction toward Earth, one at $v_1 = 0.8c$ and the other one at $v_2 = 0.85c$ relative to Earth. What is the speed of spaceship-1 as seen from the captain in spaceship-2?
- (b) Then 2 spaceships are heading toward Earth from opposite directions, at speeds of $v_3 = 0.9c$ and $v_4 = 0.85c$ relative to Earth. What is the speed of spaceship-4 as seen from the captain in spaceship-3?
- (c) If spaceship-3 is 0.3 ly ($1\text{ ly}=1\text{ lightyear}$, the distance light travels in one year) away from Earth, how much older (in seconds) will be (i) the observer, and (ii) the captain, once the spaceship-3 lands on Earth?
- (d) If the captain of spaceship-3 (proper length 450 m) instead decides to pass by Earth at its cruising speed (v_3), how long will spaceship-3 be as measured by the observer on Earth?