Formula Sheet Exam 1-3

Thermodynamics and Kinetic Theory:
$\Delta U = Q + W, \;\; pV = nRT, \;\; \bar{K} = \frac{3}{2}k_B T = \frac{1}{2}mv_{\rm rms}^2, \;\; U = \frac{3}{2}Nk_BT, \;\; k_B = R/N_A$

Lorentz transformation along x-direction

$$x' = \gamma(x - vt) \; , \;\; t' = \gamma(t - \frac{v}{c^2}x) \; , \;\; \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \; , \;\; u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \; , \;\; u'_y = \frac{u_y}{\gamma(1-\frac{u_x v}{c^2})}$$

Time dilation and length contraction:
$\Delta t' = \gamma \Delta t \; , \;\; L' = L/\gamma \quad (\Delta t, \, L$: "proper" time, length)

Relativistic Doppler effect : $\;\; f' = f\sqrt{\frac{1+v/c}{1-v/c}} \; , \;\; c = f\lambda = f'\lambda'$

Relativistic 3-momentum and energy

$$\vec{p} = \gamma m \vec{u} \; , \; E = \gamma mc^2 = \sqrt{p^2c^2 + m^2c^4} \; , \; K = E - mc^2 \; , \; \vec{u} = \frac{\vec{p}}{E}c^2 \; , \; M_{\rm bound} = m_1 + m_2 + E_B$$

Total energy and momentum conservation: $E_{\rm in} = E_{\rm out} \; , \quad \vec{P}_{\rm in} = \vec{P}_{\rm out}$

Thomson experiment: $\frac{e}{m} = \frac{V\tan\Theta}{B^2 dl}$

Blackbody: $I = P/A = \sigma T^4 \;\; , \lambda_{\rm max}T = const \;\; (const = 2.9\cdot 10^{-3} mK) \; ,$

$$u(f,T) = 8\pi h \frac{f^3}{c^3}\frac{1}{\exp(hf/k_BT) - 1} \; , \; E_\gamma = hf \; , \; c = f\lambda$$

Photoeffect: $K_{\rm max} = |eV_s| \;\; , \;\; K_{\rm max} = hf - \phi \;\; , \;\; K = \frac{1}{2}m_e v_e^2$

Compton effect: $\lambda' - \lambda = \lambda_e(1 - \cos\Theta)$

Rutherford scattering: $b = \frac{Z_1Z_2ke^2}{2K_1\tan(\theta/2)} \; , \; f = nt\,\pi b^2(\theta) \; , \; \frac{dN}{dA} \equiv N(\theta) = \frac{N_1 n_2 t}{16R^2\sin^4(\theta/2)}\left(\frac{ke^2Z_1Z_2}{K_1}\right)^2$

Bohr Model: $rm_ev_e = n\hbar \; , \; E_e = K_e + U_e = \frac{1}{2}U_e \; , \; U_e = -\frac{ke^2}{r} \; , \; r_n = n^2a_0 \; , \; a_0 = \frac{\hbar^2}{km_ee^2} \; ,$

$$E_n = -\frac{1}{n^2}\frac{ke^2}{2a_0} = -\frac{E_1}{n^2}, \; E_\gamma = E_1\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right), \; \frac{1}{\lambda} = R\left|\frac{1}{n_i^2} - \frac{1}{n_f^2}\right|, \; \mu_e = \frac{m_e}{1 + m_e/m_p}$$

X-Ray spectra: $E_\gamma = (Z-1)^2E_1(1/n_i^2 - 1/n_f^2)$

de Broglie wavelength: $\lambda = \frac{h}{p}$

Phase and group velocity: $v_{\rm ph} = \frac{\omega}{k} = c\sqrt{1 + \left(\frac{mc}{p}\right)^2} \; , \;\; v_{\rm gr} = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{c^2}{v_{\rm ph}}$

Quantum mechanical probability density: $P(x,t) = |\psi(x,t)|^2$

Uncertainty principle: $\Delta x \; \Delta p \gtrsim \frac{\hbar}{2} \;\; , \;\; \Delta t \; \Delta E \gtrsim \frac{\hbar}{2}$

Gaussian wavepackage: $|\psi(x)|^2 = \frac{1}{\sqrt{2\pi}\alpha}\exp(-\frac{x^2}{2\alpha^2})$

Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + U(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$

time part: $i\hbar\frac{\partial}{\partial t}\phi(t) = E\phi(t)$, space part: $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + U(x)\psi(x) = E\psi(x)$

Operators: $\hat{x} = x$, $\hat{p} = -i\hbar\frac{\partial}{\partial x}$, $\hat{K} = \frac{\hat{p}^2}{2m}$, $\hat{E} = i\hbar\frac{\partial}{\partial t}$

Expectation values: $\langle\mathcal{O}\rangle = \int_{-\infty}^{+\infty} dx\psi^*(x)\hat{\mathcal{O}}\psi(x)$, Uncertainty: $\Delta\mathcal{O} = \sqrt{\langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2}$

Potential energy levels:

infinite square well: $E_n = n^2\frac{\hbar^2\pi^2}{2mL^2}$,

finite square well (small n): $E_n \simeq n^2\frac{\hbar^2\pi^2}{2m(L+2\delta)^2} - U_0$ with $\delta \simeq \frac{\hbar}{\sqrt{2mU_0}}$,

simple harmonic oscillator: $E_n = (n+\frac{1}{2})\hbar\omega$ with $\omega^2 = \frac{k}{m}$

Tunneling: $T(E) = \left[1 + \frac{1}{4}\left[\frac{U_0^2}{E(U_0-E)}\right]\sinh^2(\alpha L)\right]^{-1}$ with $\alpha^2 = \frac{2m(U_0-E)}{\hbar^2}$, $\delta = \frac{1}{\alpha}$,

$\sinh x = \frac{1}{2}(e^x - e^{-x})$, $R(E) + T(E) = 1$

Barrier scattering: $T(E) = \left[1 + \frac{1}{4}\left[\frac{U_0^2}{E(E-U_0)}\right]\sin^2(k_2L)\right]^{-1}$ with $k_2^2 = \frac{2m(E-U_0)}{\hbar^2}$

Constants and Conversions

$c = 3\cdot 10^8\ m/s$, $k_B = 8.62\cdot 10^{-5}\,eV/K$, $R = 8.31 J/K\cdot mol$, $\sigma = 5.67\cdot 10^{-8}\,W/(m^2K^4)$,
$e = 1.6\cdot 10^{-19}C$, $1\ eV = 1.6\cdot 10^{-19}J$, $1\,MeV = 10^6\,eV$, $1\,GeV = 10^9\,eV$,
$1u = 1.66\cdot 10^{-27}\,kg = 931.5\,MeV/c^2$, $m_p = 938.3MeV/c^2$, $m_n = 939.6MeV/c^2$, $m_e = 511\,keV/c^2$,
$1\,fm = 10^{-15}m$, $1\,nm = 10^{-9}m$,
$h = 6.63\cdot 10^{-34}Js = 4.14\cdot 10^{-15}\,eVs$, $\hbar = h/(2\pi)$, $\lambda_e = h/(m_ec) = 0.00243\,nm$,
$hc = 1.24\cdot 10^{-6}eVm = 1240\,eVnm$, $\hbar c = 197\,eVnm = 197\,MeV fm$
$a_0 = 0.0529\,nm$, $E_1 = ke^2/(2a_0) = 13.6\,eV$, $R = 1.1\cdot 10^7m^{-1} = 1/(91.2\,nm)$, $ke^2 = 1.44\,eVnm$