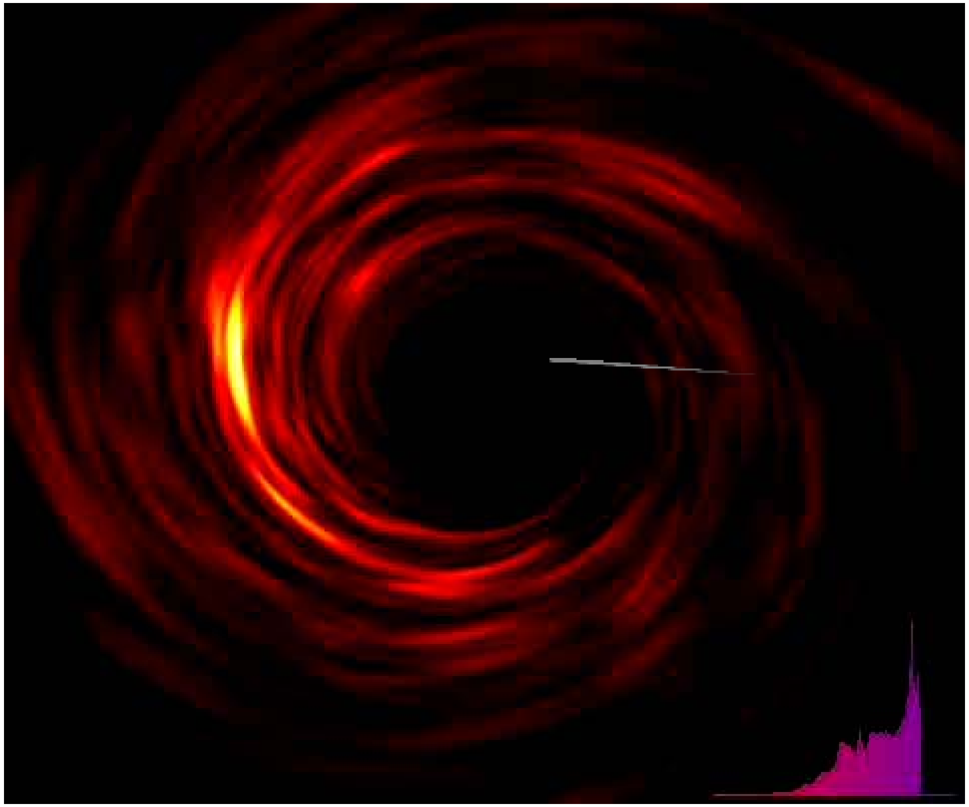
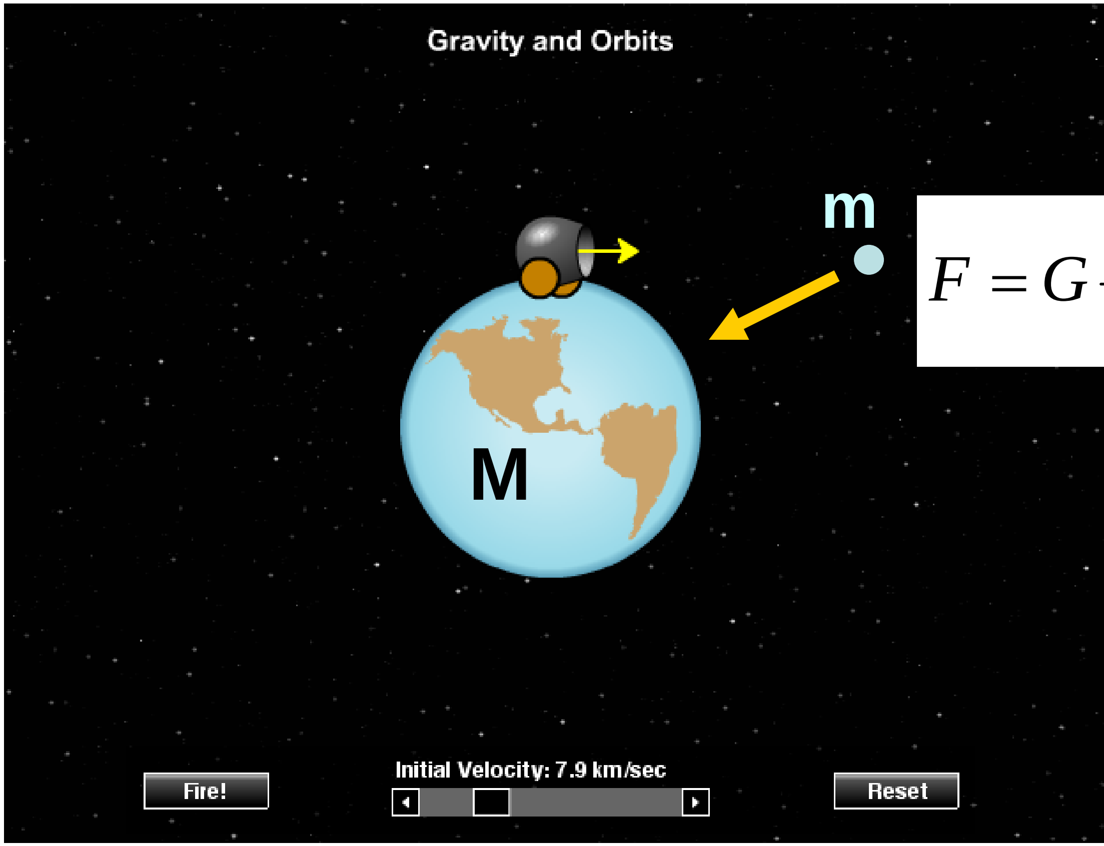
A Plunge Into a Black Hole



A black hole is a region of space-time from which nothing can escape, even light.

Gravity in Newtonian physics



$$F = G\frac{mM}{r^2}$$

PLAY ANIMATION

Escape condition:
Kinetic Energy K ≥ Gravitational Potential Energy |U|

$$K = mV^2/2; \quad U = -\frac{GMm}{R}$$

At threshold:

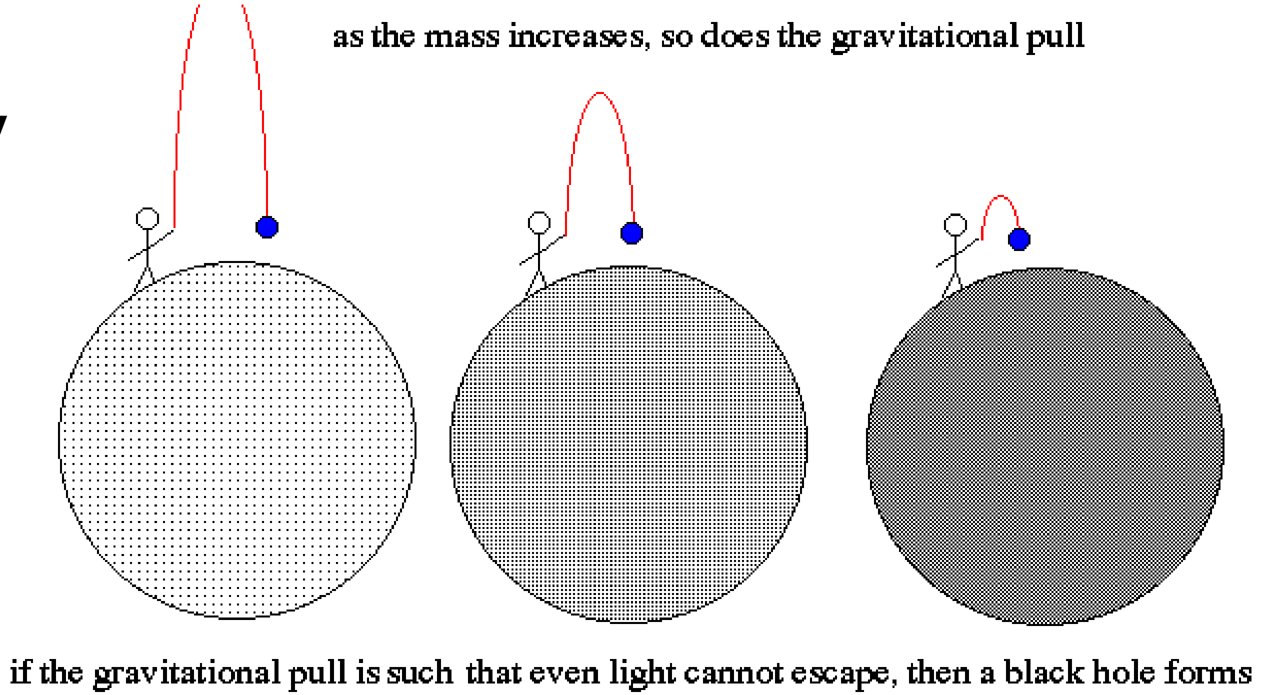
$$K = \frac{mV_{esc}^2}{2} = |U| = \frac{GMm}{R} \Rightarrow V_{esc} = \sqrt{\frac{2GM}{R}}$$

Escape velocities for some objects

Object	Mass	Escape velocity
Ceres (largest asteroid)	10^{21} kg	0.64 km/s
The Moon	7×10^{22} kg	2.38 km/s
The Earth	6×10^{24} kg	11.2 km/s
Jupiter	2×10^{27} kg	60 km/s
The Sun	2×10^{30} kg	618 km/s

What happens with even more massive and dense objects?

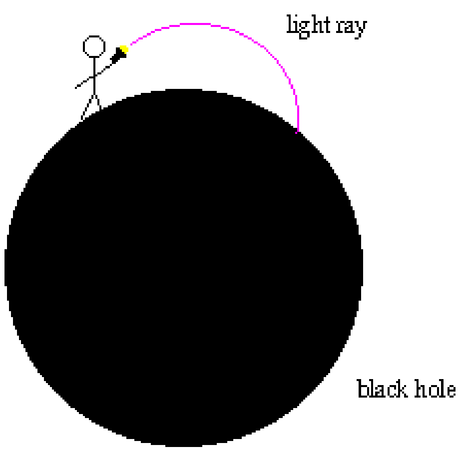
Black holes in Newtonian physics



First suggested by Laplace in 1796

Critical (Schwarzschild) radius

$$V_{esc} = \sqrt{\frac{2GM}{R_s}} = c \Rightarrow R_s = \frac{2GM}{c^2}$$



The result is accidentally correct, but derivation is wrong and picture is wrong. We need general relativity!

Newton's theory is a weak-gravity limit of a more general theory: General Relativity

Even in the weak gravity of the Earth and the Sun, there are measurable deviations from Newtonian mechanics and gravitation law!

- Advance of Mercury's perihelion
- Bending of light by the Sun's gravity

General Relativity predicts new effects, completely absent in the Newton's theory: black holes, event horizons, gravitational waves.

General Relativity

Albert Einstein

Developed in 1907-1915 by A. Einstein in close collaboration with mathematicians: Grossmann, Hilbert, Levi-Civita

... in all my life I have not laboured nearly so hard, and I have become imbued with great respect for mathematics, the subtler part of which I had in my simple-mindedness regarded as pure luxury until now.

Marcel Grossmann

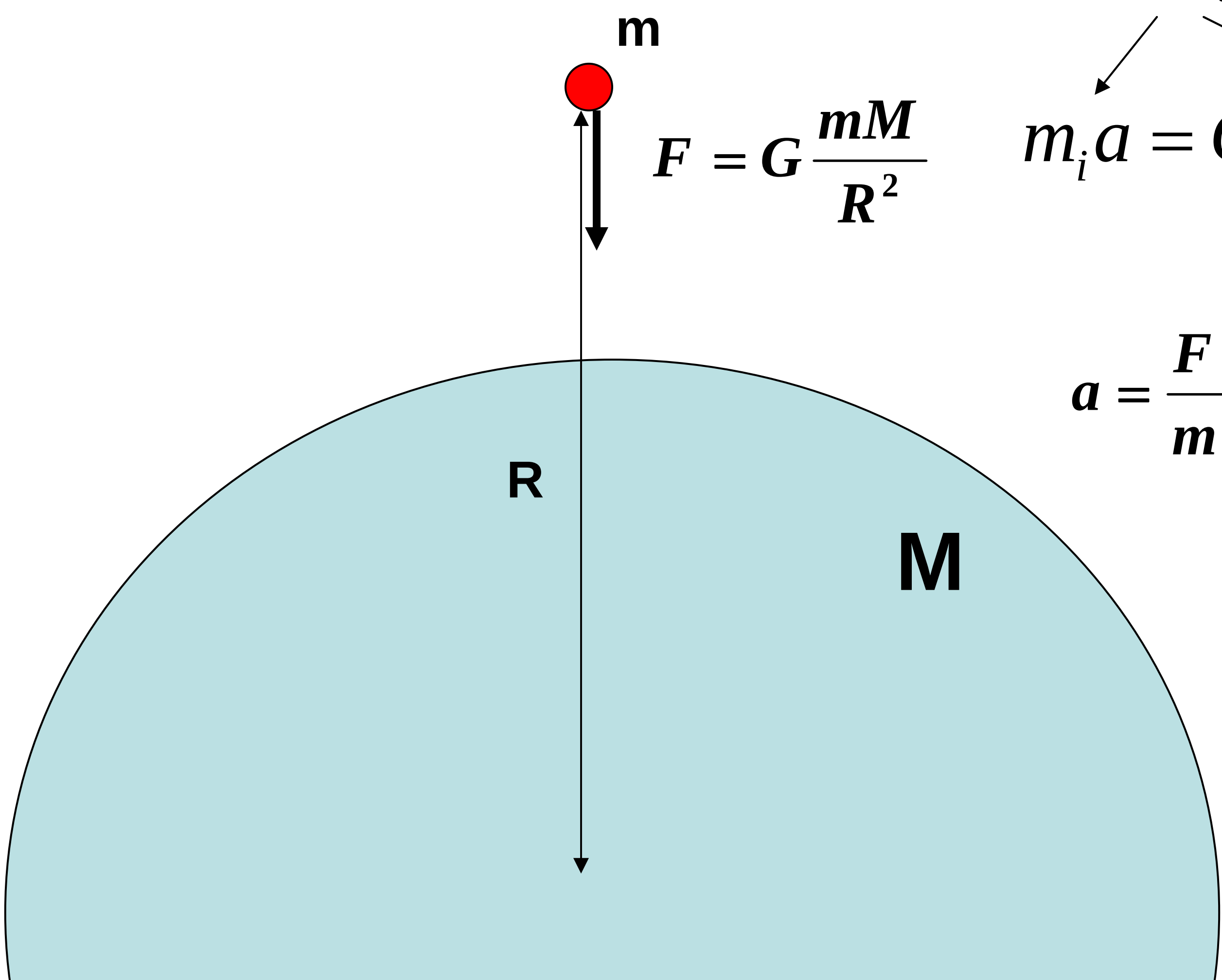
David Hilbert

Tullio Levi-Civita

Gravity is a strange force. It has a unique property:

All bodies in the same point in space experience the same acceleration!

Galileo, about 1600

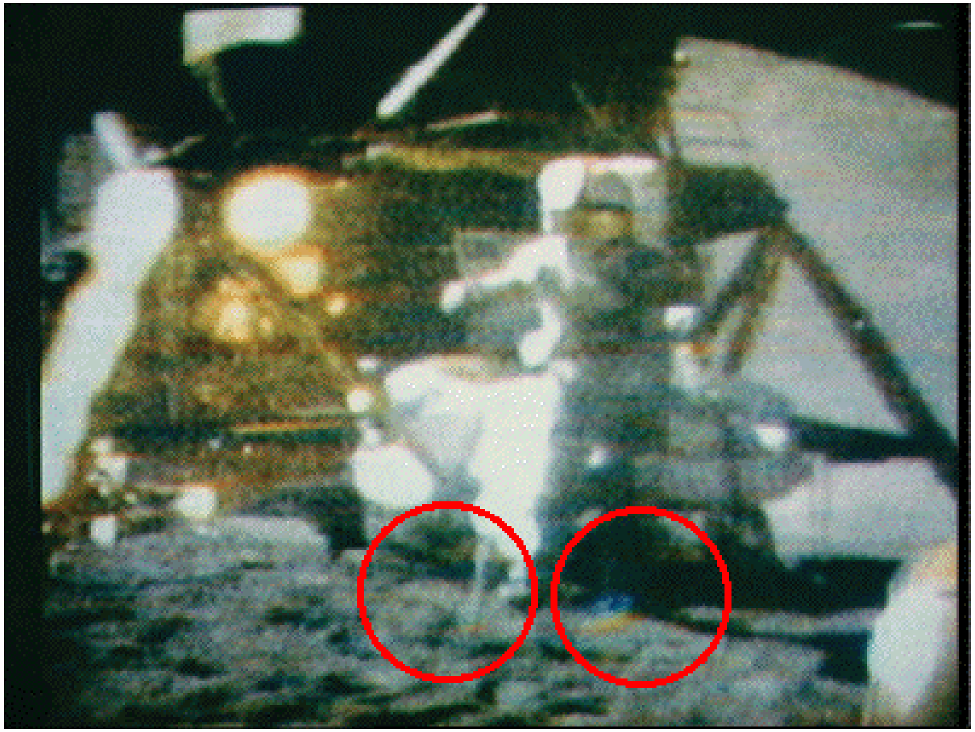


$$F = G\frac{mM}{R^2}$$

$$m_i = m_g \;!!!$$

$$m_i a = G\frac{m_g M}{R^2}$$

$$a = \frac{F}{m} = G\frac{M}{R^2}$$



Apollo 15
August 2 1971

Astronaut David Scott (Apollo 15 commander) watches as a geological hammer, released from his right hand and a feather, released from his left hand at the same instant, hit the lunar surface simultaneously, in a test of Galileo's law of motion concerning falling objects.

Equivalence Principle

In 1907, Einstein was preparing a review of special relativity when he suddenly wondered how Newtonian gravitation would have to be modified to fit in with special relativity. At this point there occurred to Einstein, described by him as the *happiest thought of my life* , namely that an observer who is falling from the roof of a house experiences no gravitational field. He proposed the *Equivalence Principle* as a consequence:-

... we shall therefore assume the complete physical equivalence of a gravitational field and the corresponding acceleration of the reference frame. This assumption extends the principle of relativity to the case of uniformly accelerated motion of the reference frame.

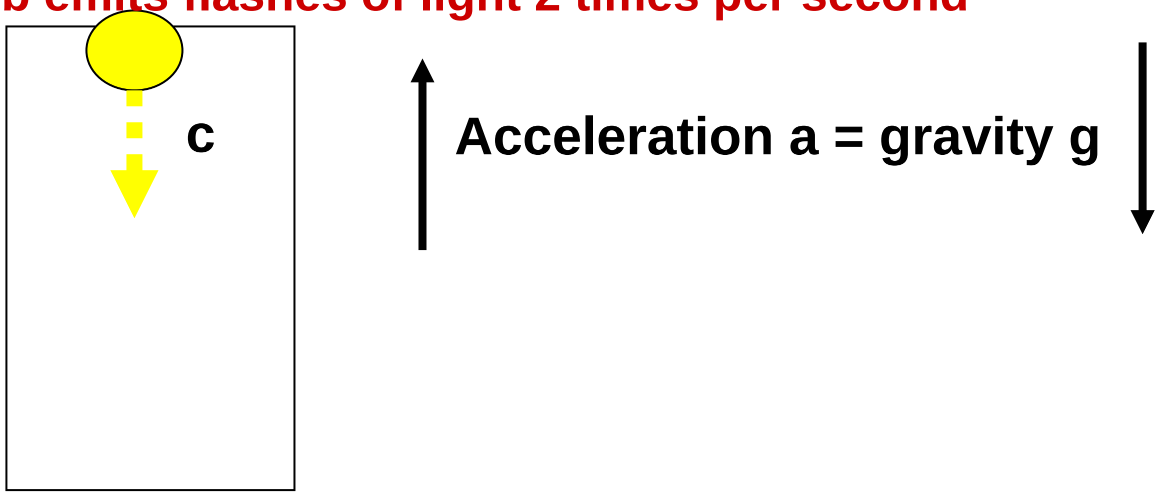
This means that in the freely-falling elevator cabin you don't feel any effects of gravity! You and all objects around you experience the same acceleration.

Vice versa: in outer space you can imitate the effect of gravity by acceleration.

Immediate consequences of the Equivalence Principle:

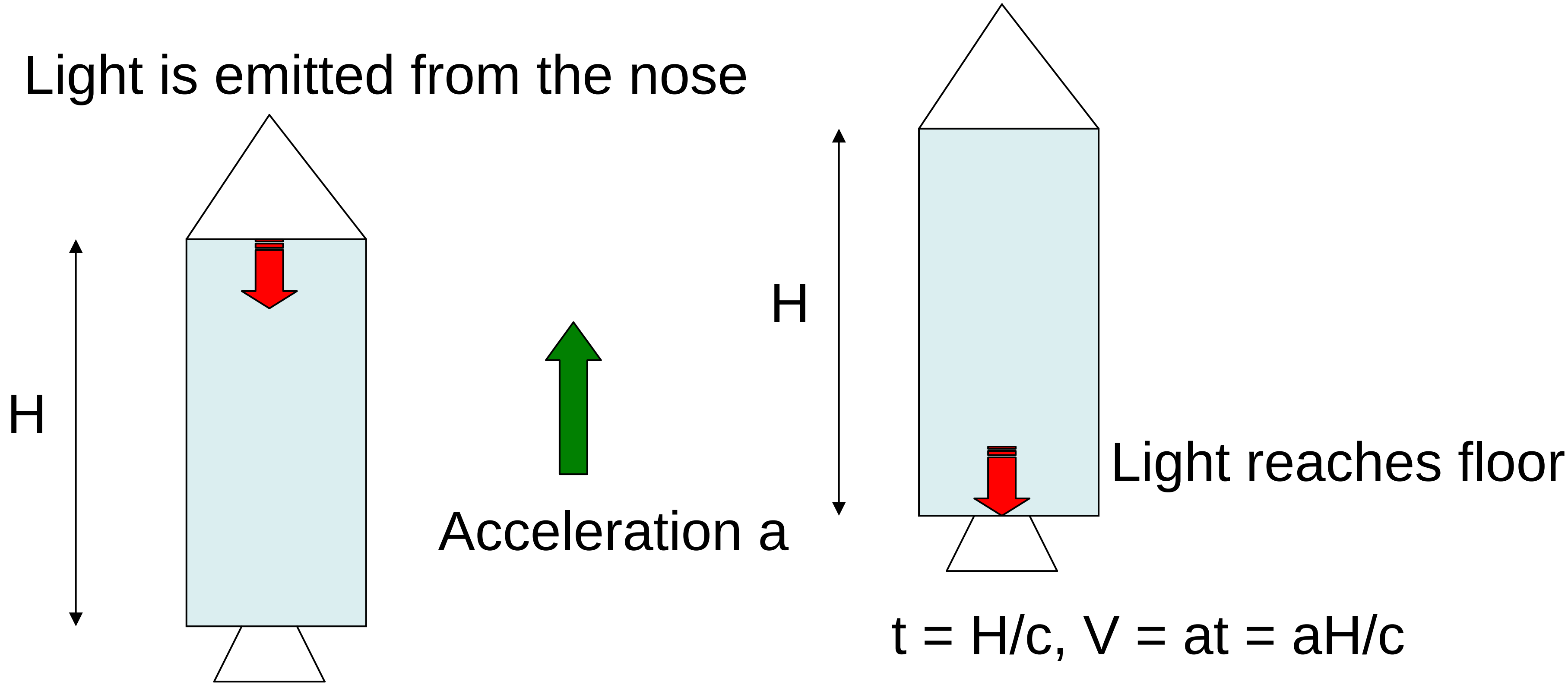
- Bending of light in the gravitational field
- Time flow and frequency of light are changed in the gravitational field

The bulb emits flashes of light 2 times per second



An observer on the floor receives flashes faster than 2 times per second

Frequency of light is shifted in the accelerated frame. It should be also shifted in the gravitational field!

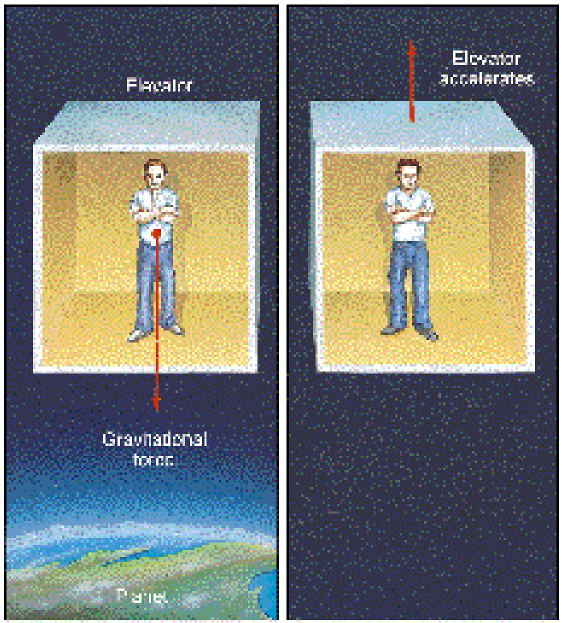


t = 0, V = 0

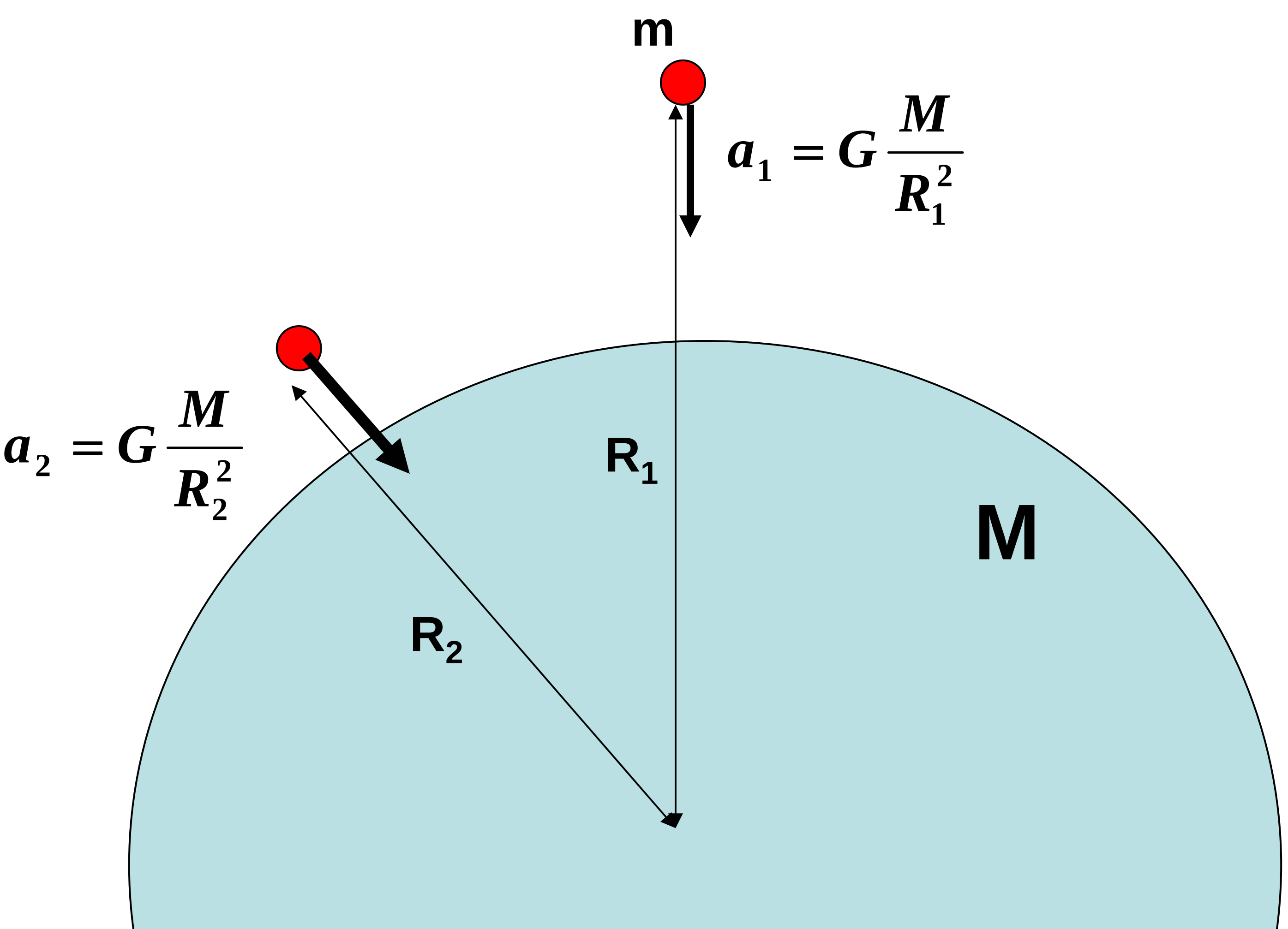
Doppler effect: $\frac{\Delta f}{f} = \frac{V}{c} = \frac{aH}{c^2}$

First observed on the Earth by Pound and Rebka 1960: relative frequency shift of 10^{-15} over the height of 22 m.

Light should be bent in the gravitational field



Warning: all bodies experience the same acceleration, but only in a small region of space. In another region this acceleration is different. Time flows with a different rate, and paths are bent differently in these two regions.



If gravity can be eliminated or imitated by motion, no special force of gravity is needed!

The force of gravity is actually the acceleration you feel when you move through space-time

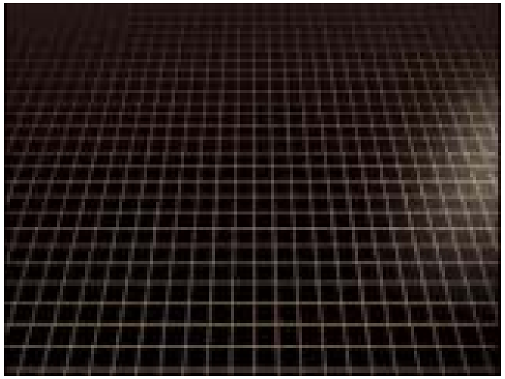
How to explain that in the absence of any force the trajectories are not straight lines?

Because space and time are curved by the matter!

Main idea:

Space-time gets curved by masses. Objects traveling in curved space-time have their paths deflected, as if a force has acted on them.

“Curvature” of time means that the time flows with a different rate in different points in space



"Matter tells spacetime how to bend and spacetime returns the compliment by telling matter how to move."

John Wheeler

Aristotle: there are absolute space and absolute time. There is absolute rest different from motion for all observers.

Galileo-Newton: there are absolute space and absolute time. Motion and rest are relative to an observer. Laws of physics are the same for all uniformly moving observers. Acceleration is absolute.

Einstein (Special relativity): space and time are relative to an observer. Laws of physics are the same for all uniformly moving observers. Space-time is a fixed flat background.

Einstein (General relativity):

- ALL kinds of motion are unified, including accelerated motion
- Gravity and acceleration are unified and depend on the observer
- Space-time is not a fixed background anymore. Space-time and matter interact with each other and affect each other.

$$\mathbf{G} = 8\pi\mathbf{T}$$

Describes curvature of space-time

Mass-energy density of matter

About 1912 Einstein realized that the geometry of our world should be non-Euclidean.

He consulted his friend Grossmann who was able to tell Einstein of the important developments of Riemann, Ricci and Levi-Civita.

G.F.B. Riemann
(1826-1866)

When Planck visited Einstein in 1913 and Einstein told him the present state of his theories Planck said:

As an older friend I must advise you against it for in the first place you will not succeed, and even if you succeed no one will believe you.

Several versions of Einstein's GR in 1913-1914 were wrong.

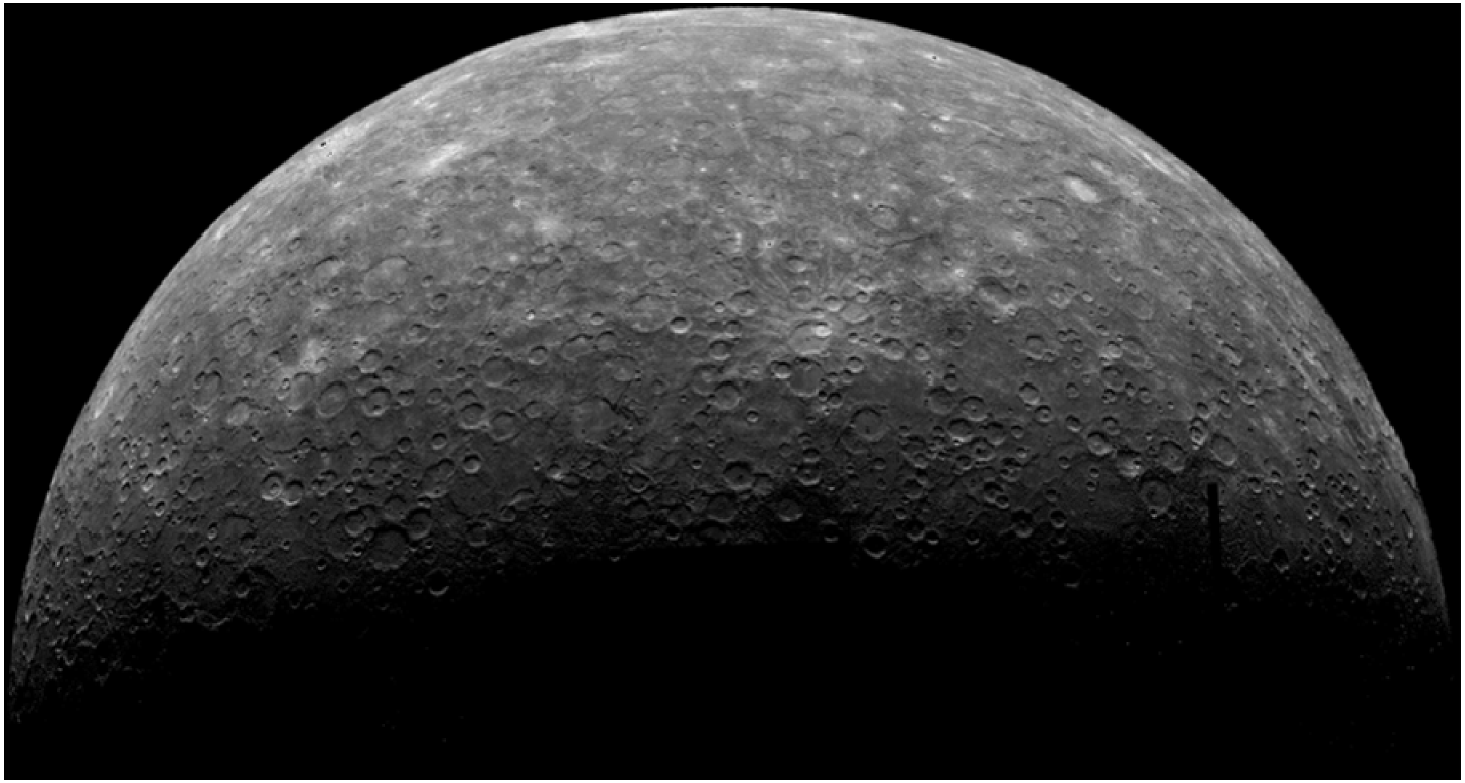
Only in November 1915, after correspondence with Levi-Civita and Hilbert, Einstein published a paper with correct equations.

Hilbert also published correct equations, in fact 5 days earlier than Einstein.

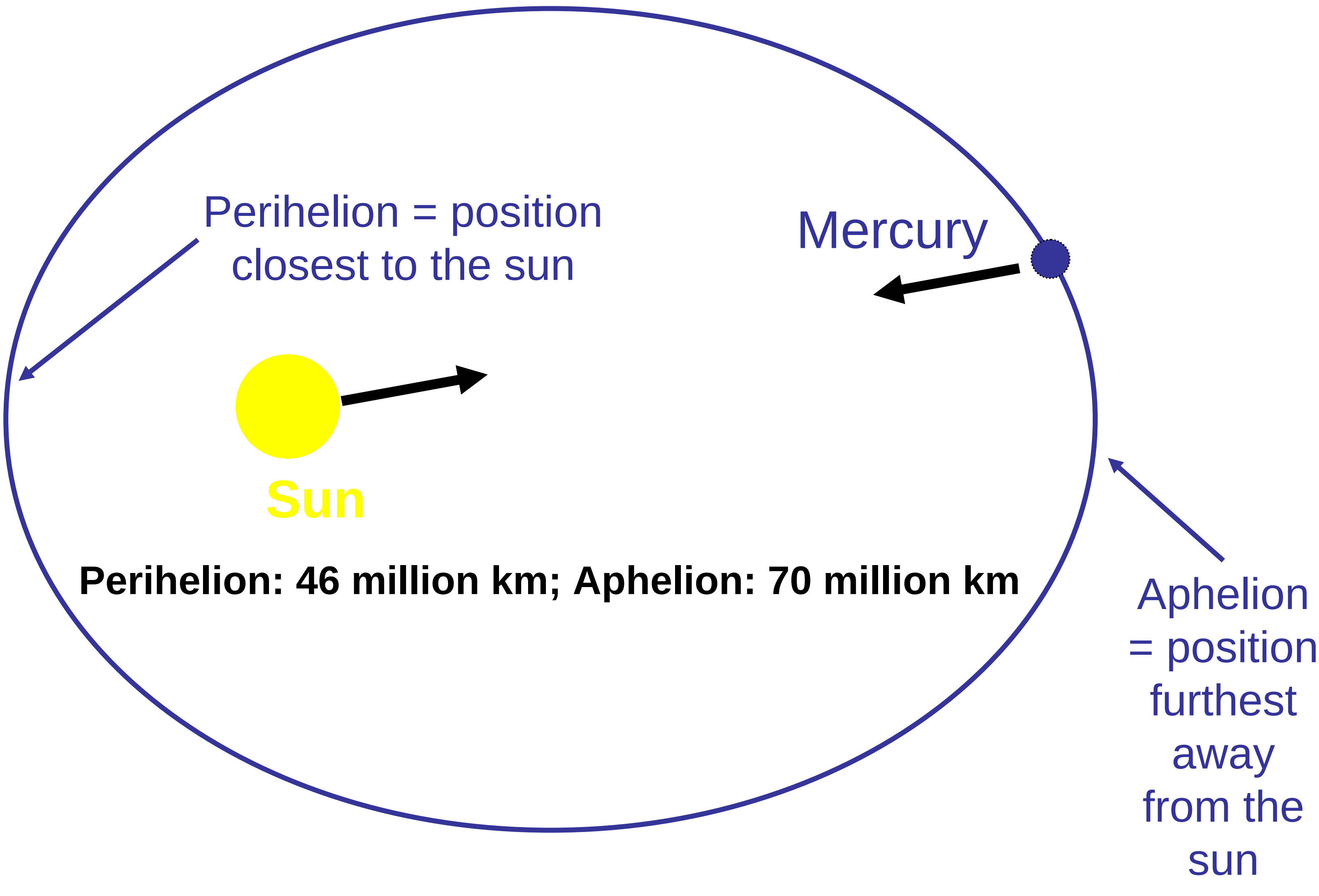
On the 18th November Einstein made a discovery about which he wrote *For a few days I was beside myself with joyous excitement* . He explained the advance of the perihelion of Mercury with his theory.

One little speck on the brilliant face of Newton's theory:

The advance of the perihelion of Mercury



Mercury: the closest planet to the Sun

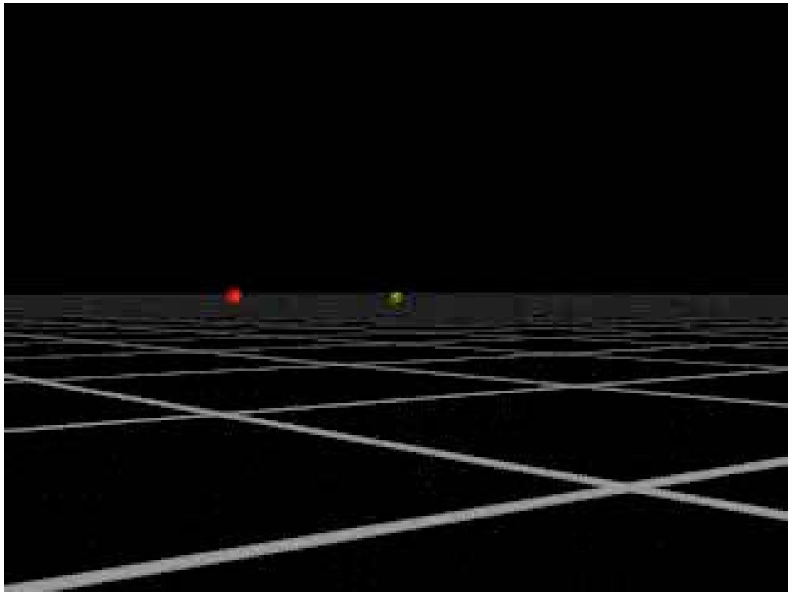


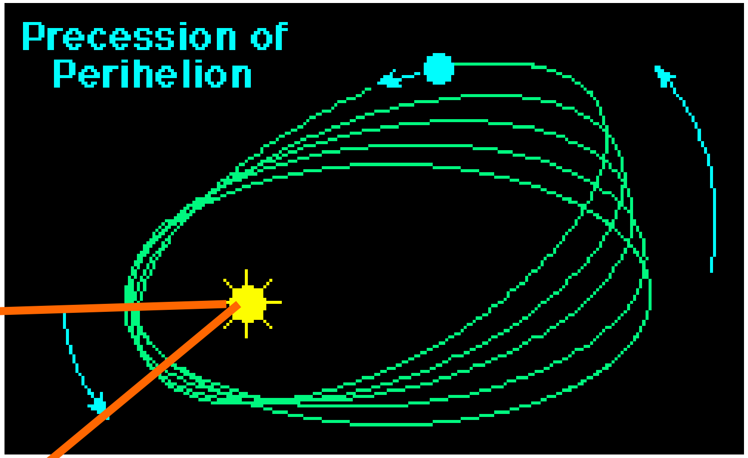
In reality the orbits deviate from elliptical:

Mercury's perihelion precession: 5600.73 arcseconds per century

Newtonian perturbations from other planets: 5557.62 arcseconds per century

Remains unexplained: 43 arcseconds/century (Le Verrier 1855)





1 degree = 3600 arcseconds

Predicted the presence and position of Neptune from irregularities in Uranus's orbit

Neptune was found in 1846 exactly at the predicted position

Urbain Le Verrier 1811-1877

In the eyes of all impartial men, this discovery [Neptune] will remain one of the most magnificent triumphs of theoretical astronomy …

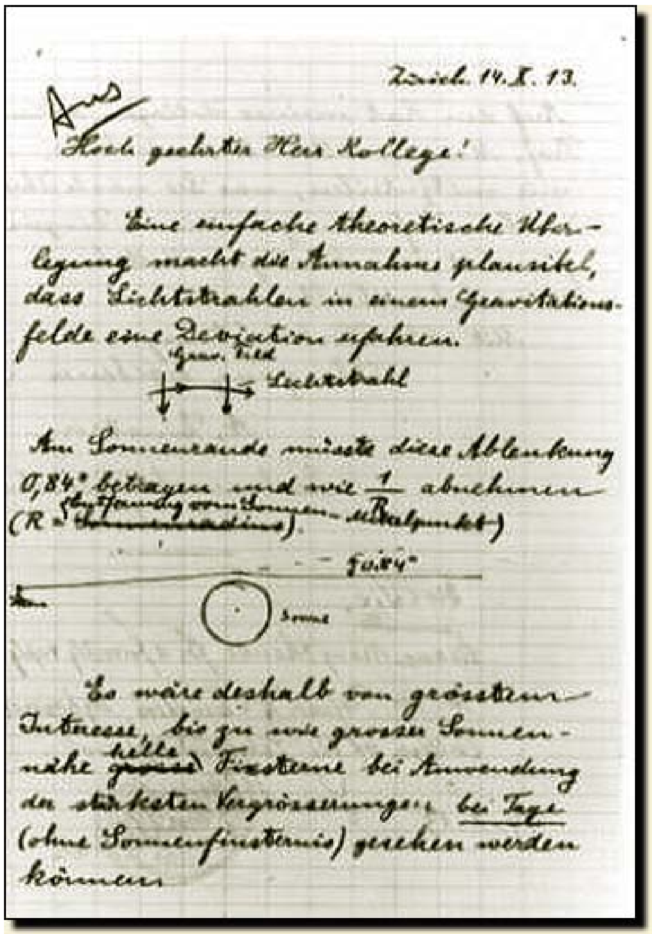
Arago

I do not know whether M. Le Verrier is actually the most detestable man in France, but I am quite certain that he is the most detested.

A contemporary

In 1855 Le Verrier found that the perihelion of Mercury advanced slightly more than the Newtonian theory predicted.
He and others tried to explain it with a new planet Vulcan, new asteroid belt, etc.
Finally, GR provided an explanation.

Bending of light: triumph of GR



Zürich. 14. X. 13.

Hoch geehrter Herr Kollege!

Eine einfache theoretische Überlegung macht die Annahme plausibel, dass Lichtstrahlen in einem Gravitationsfelde eine Deviation erfahren.

Grav. feld

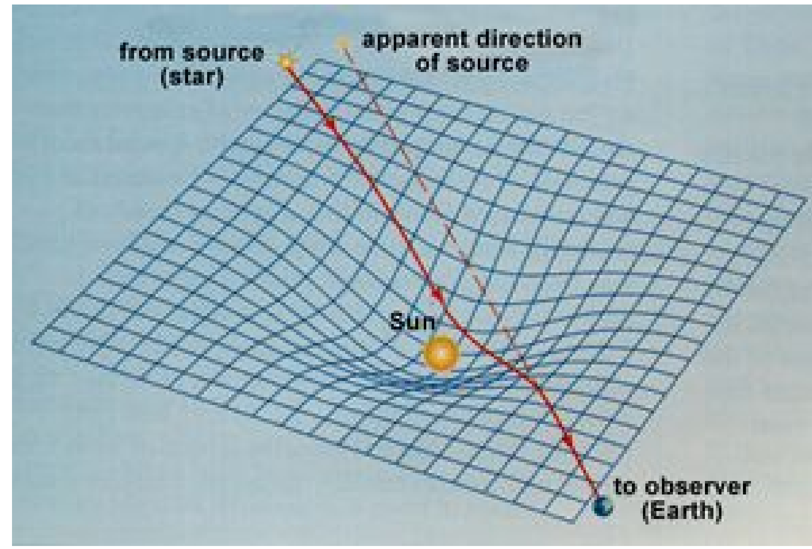
Lichtstrahl

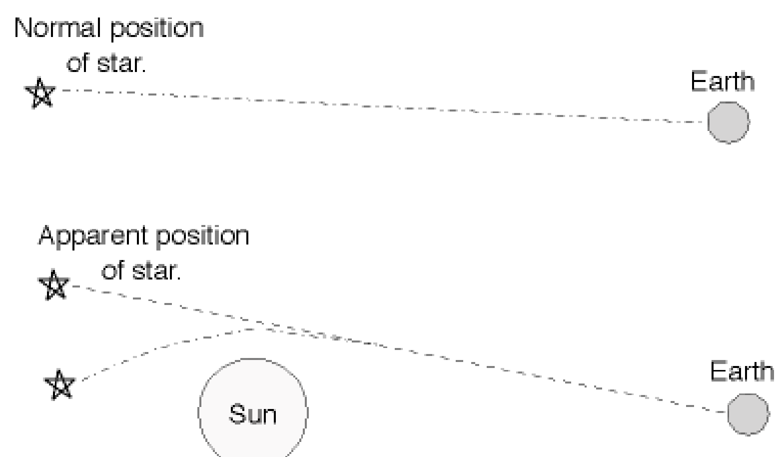
Am Sonnenrande müsste diese Ablenkung 0,84" betragen und wie $\frac{1}{R}$ abnehmen (R = Abstand vom Sonnen-Mittelpunkt).

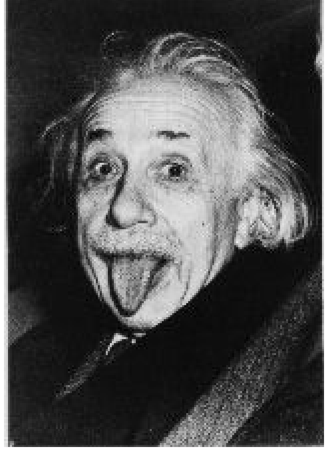
0,84"

Sonne

Es wäre deshalb von grösstem Interesse, bis zu wie grosser Sonnennähe helle Fixsterne bei Anwendung der stärksten Vergrösserungen bei Tage (ohne Sonnenfinsternis) gesehen werden können.

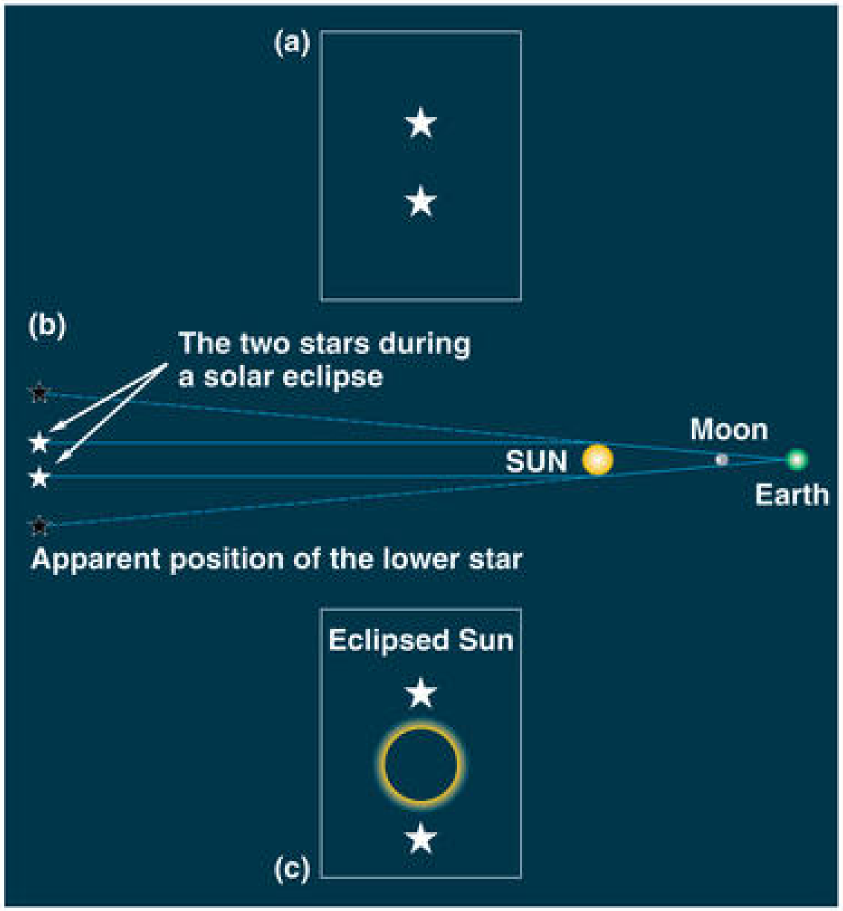


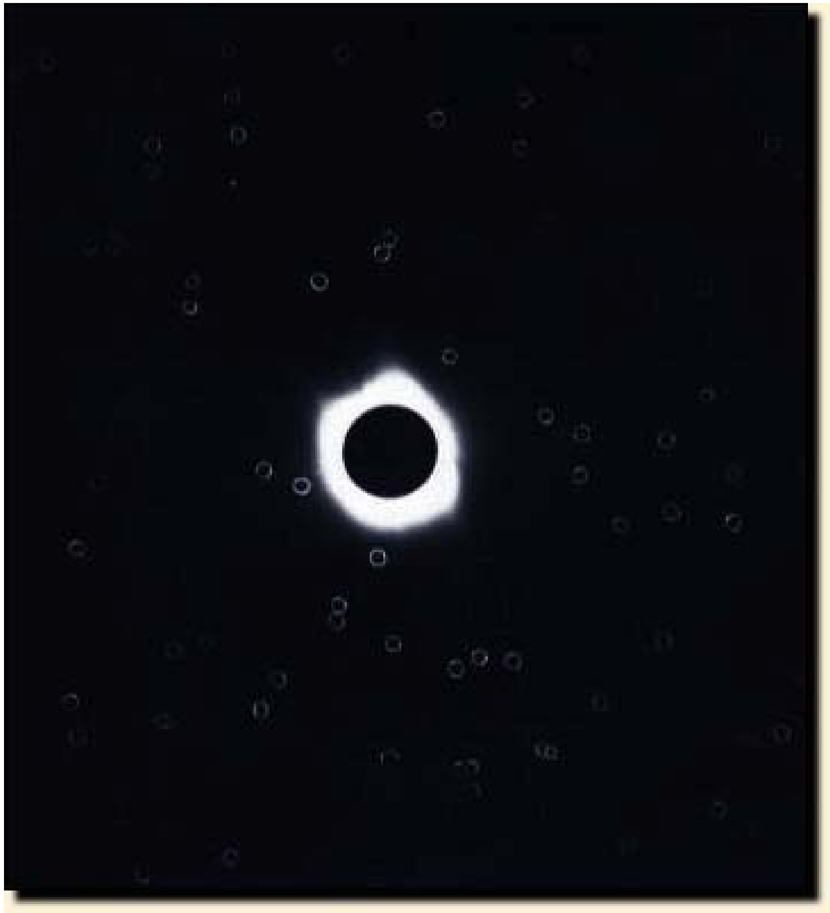




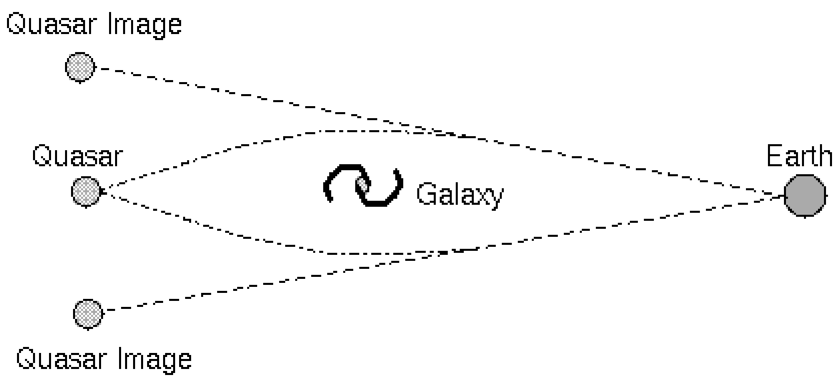
Two British expeditions in 1919 confirmed Einstein's prediction.

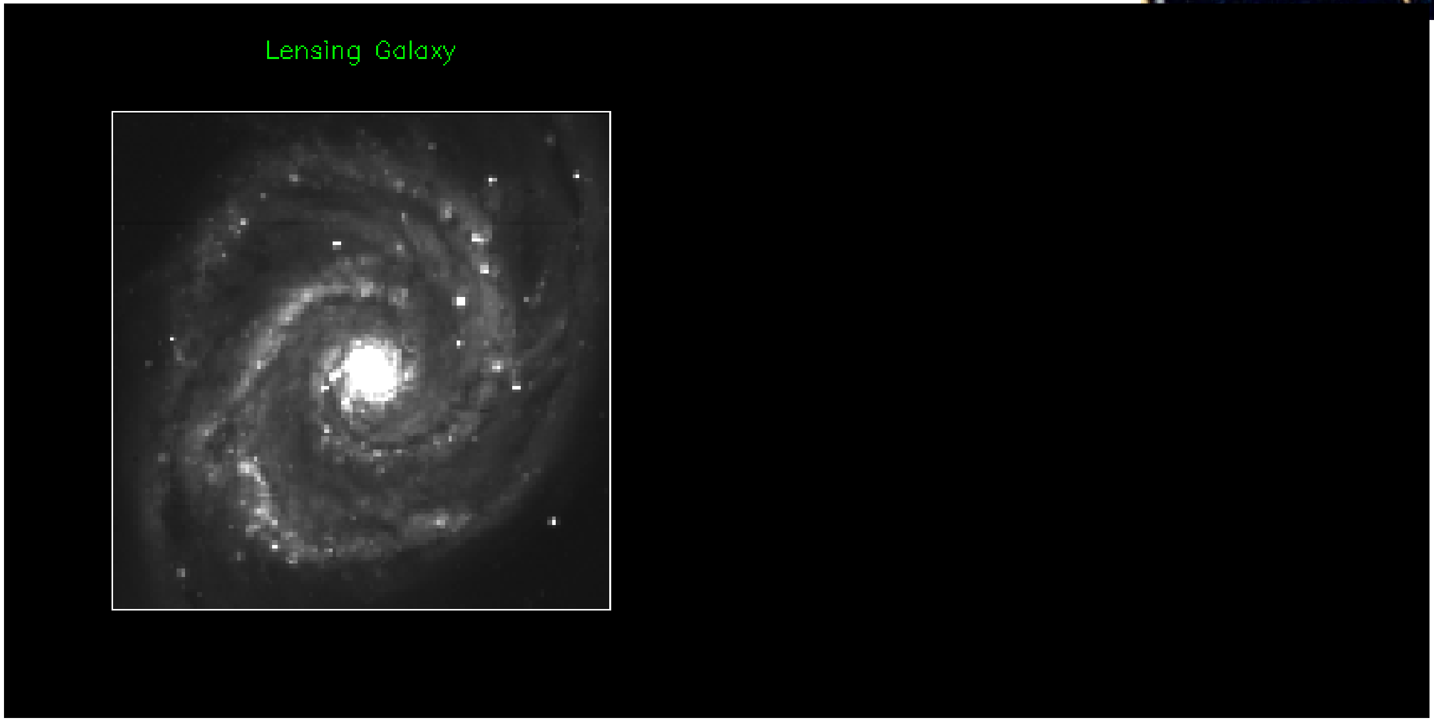
The shift was about 1.74 seconds of arc, as predicted!



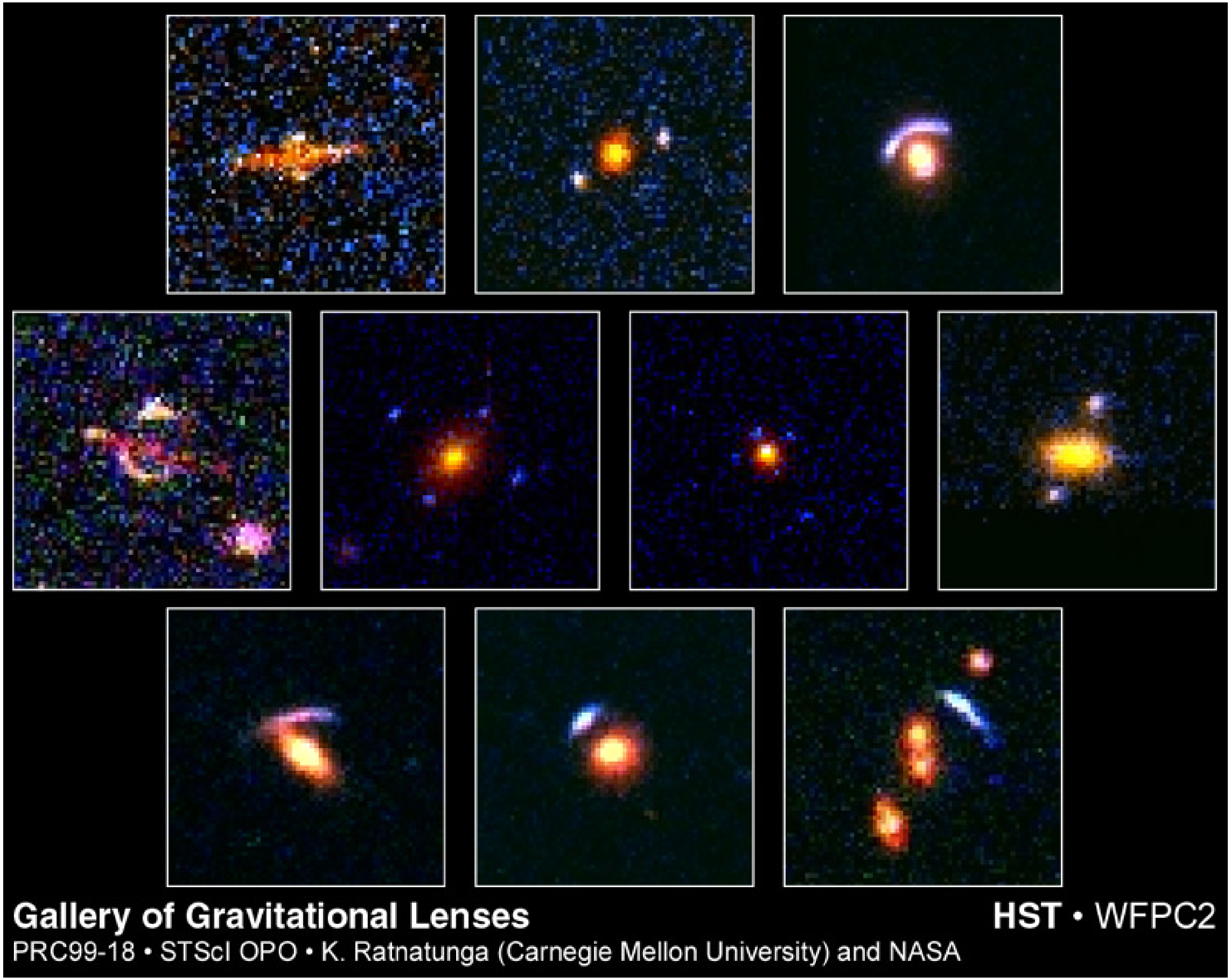


Gravitational lensing



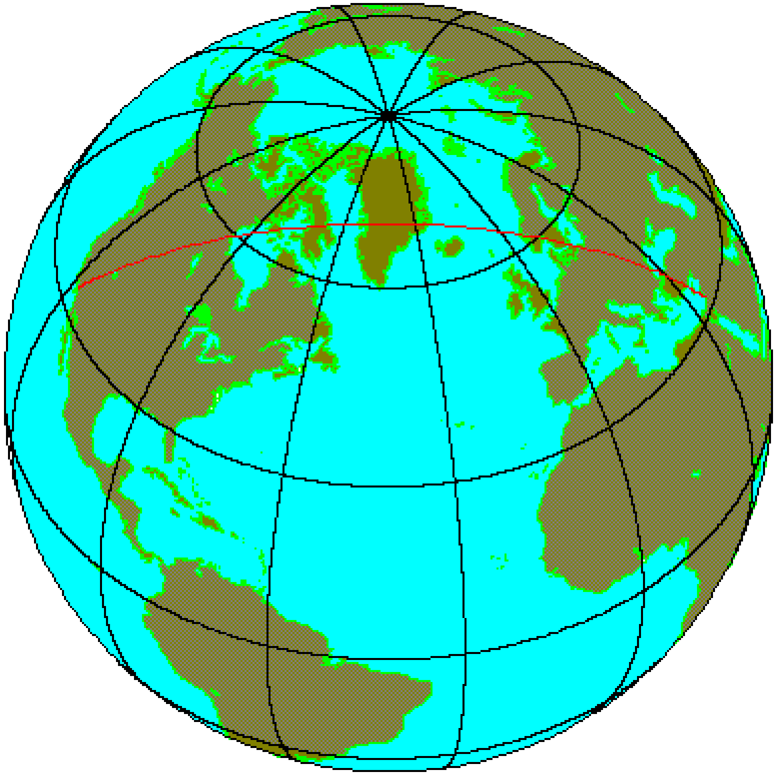


Gallery of lenses (Hubble Space Telescope)

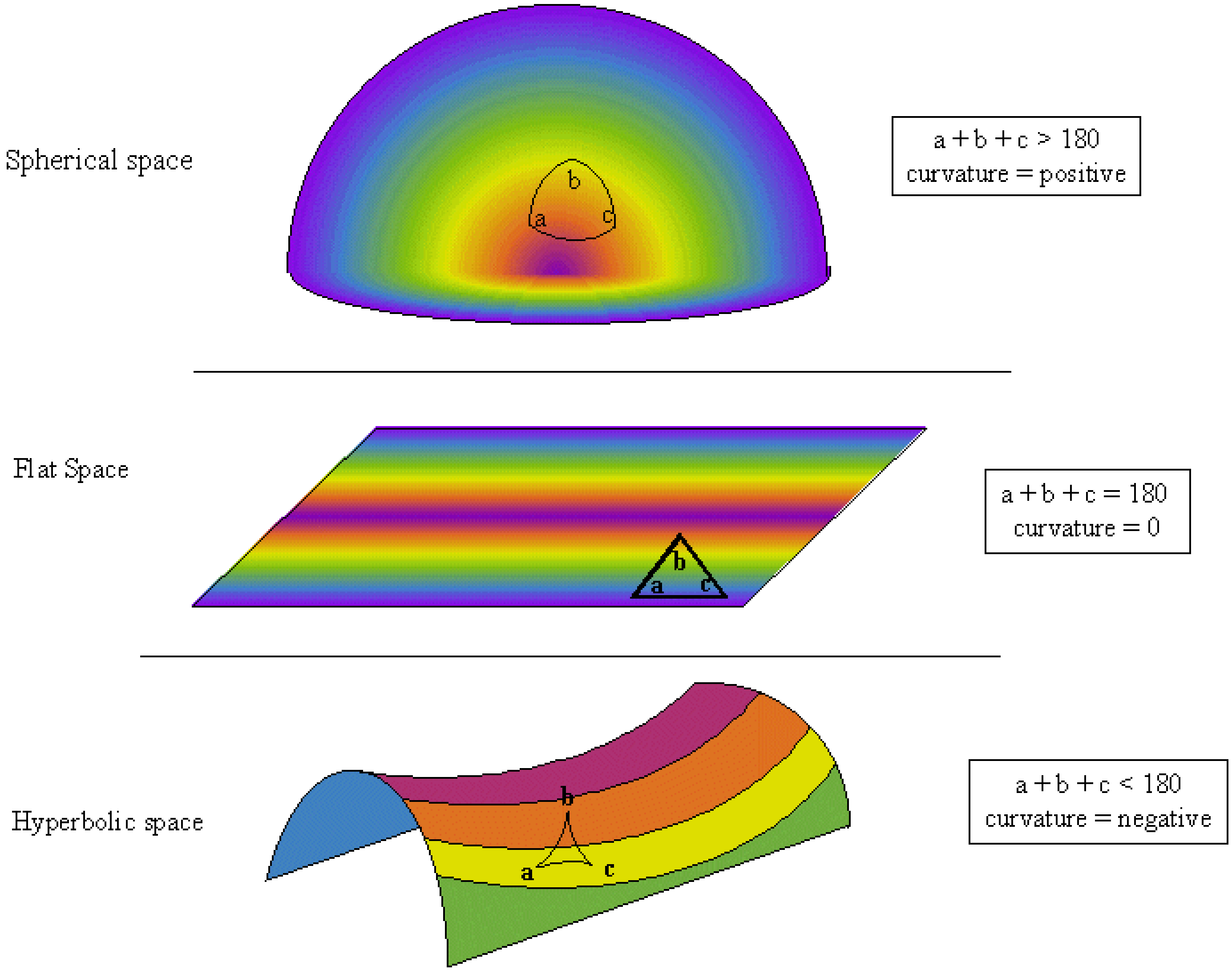


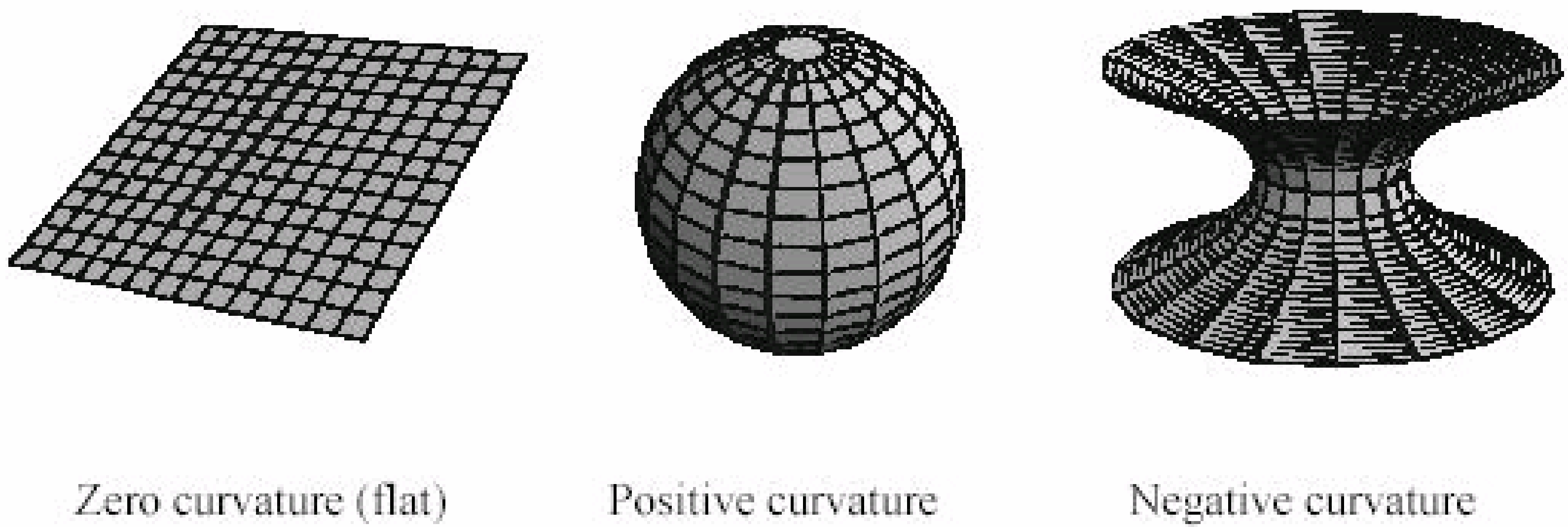
Curved space

Shortest paths are called geodesics; they are not straight lines!

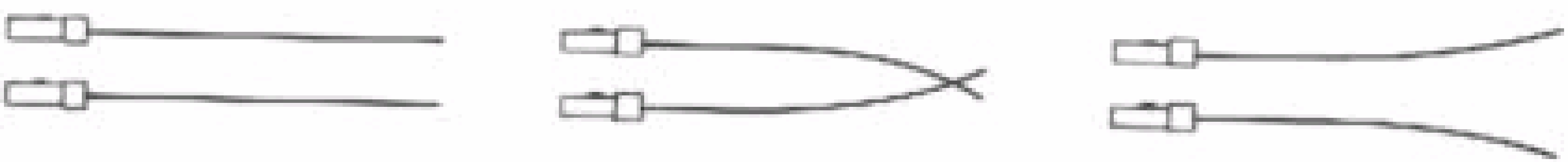


The shortest path between two cities is not a straight line

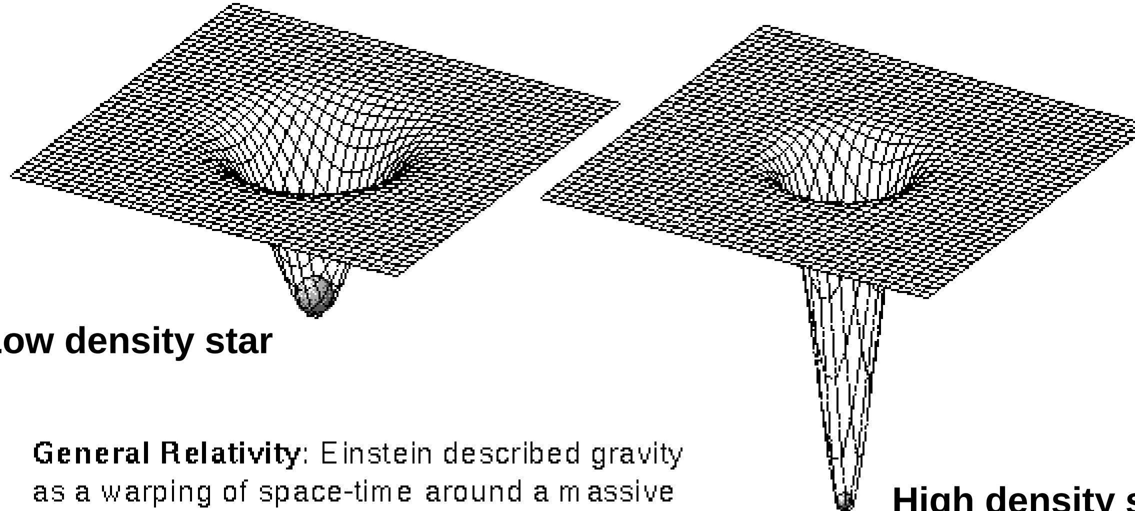

Spherical space
b
a
c
a + b + c > 180
curvature = positive
Flat Space
b
a
c
a + b + c = 180
curvature = 0
Hyperbolic space
b
a
c
a + b + c < 180
curvature = negative



Light rays:

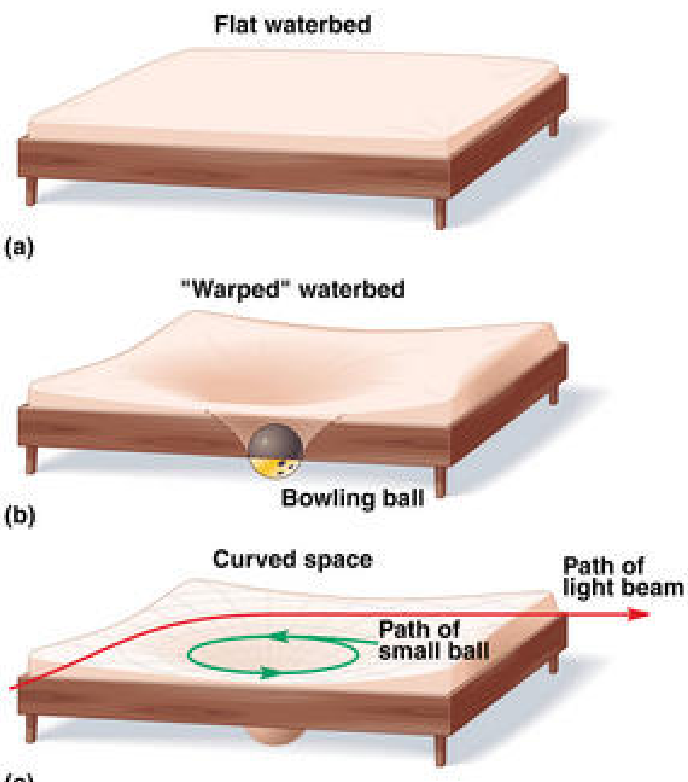


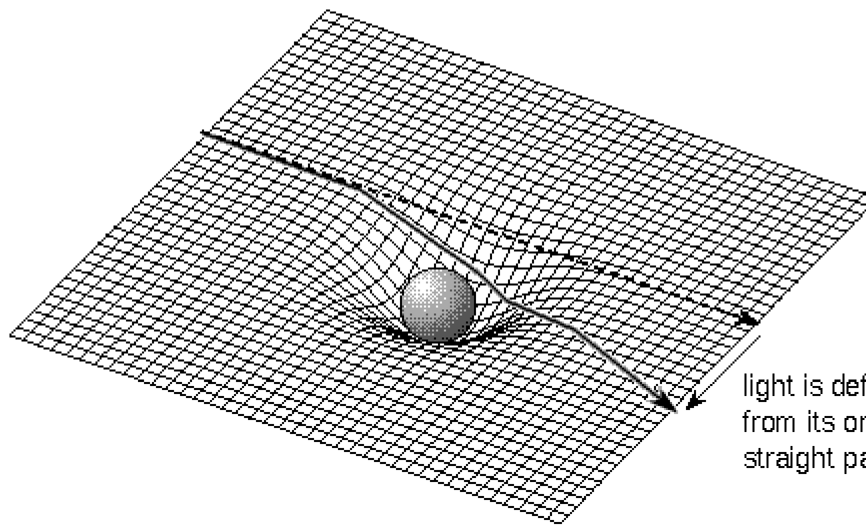
parallel **converge** **diverge**



General Relativity: Einstein described gravity as a warping of space-time around a massive object. The stronger the gravity, the more space-time is warped.

Embedding diagrams





General Relativity: Light travels along the curved space taking the shortest path between two points. Therefore, light is deflected toward a massive object! The stronger the local gravity is, the greater the light path is bent.

Riemann geometry of space-time

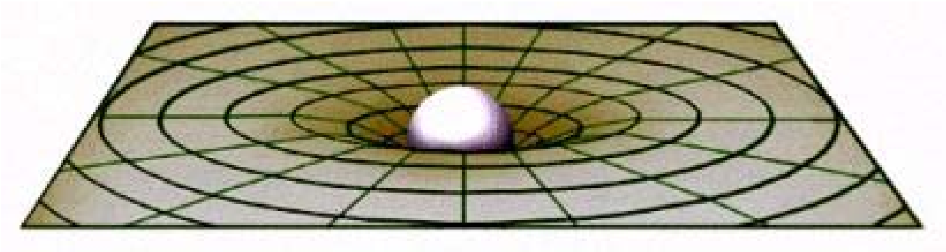
G.F.B. Riemann
(1826-1866)

Metric: interval *ds* between two close points as measured by a local observer

Metric outside a spherically symmetric body:

K. Schwarzschild 1916

$$ds^2 = \frac{dr^2}{1-\frac{R_s}{r}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - \left(1-\frac{R_s}{r}\right)dt^2$$



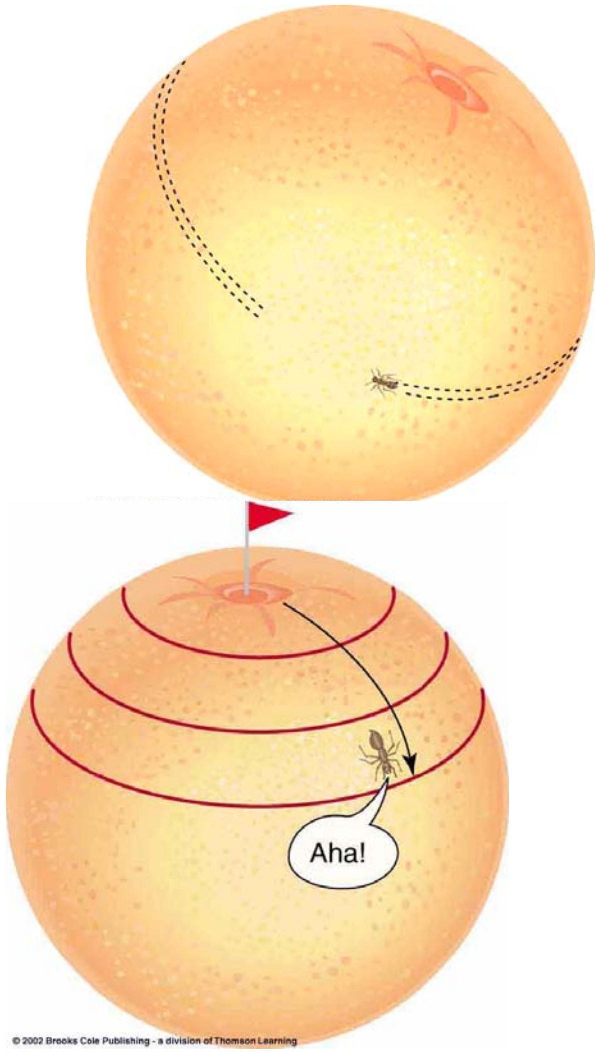
$$R_s = \frac{2GM}{c^2}$$

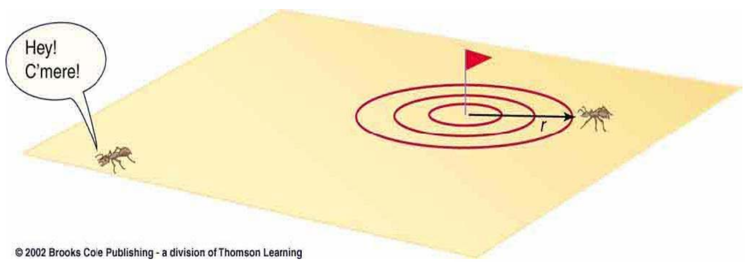
2D cross-section of 4D space-time at t = 0, θ = π/2

Determining geometry of space-time

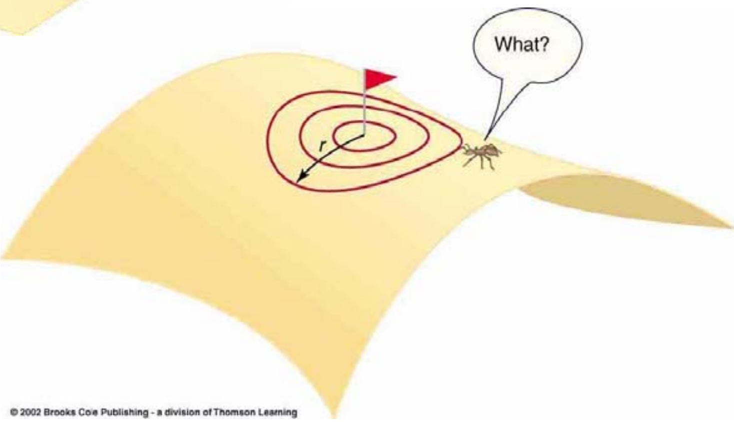
How can a 2-D creature investigate the geometry of the sphere?

Measure curvature of its space!





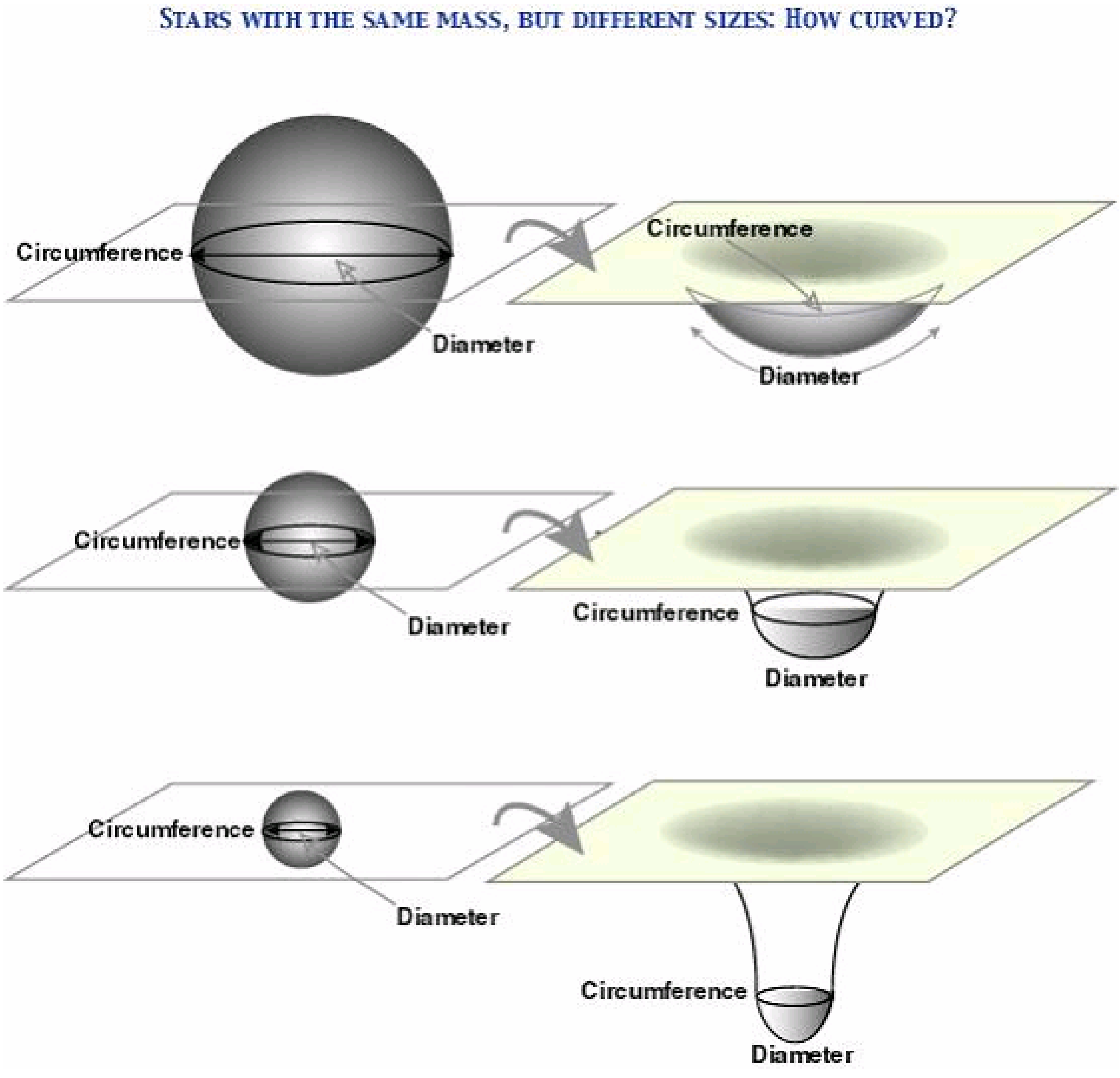
Flat surface (zero curvature)



positive curvature = $1/R^2$

negative curvature

Back to black holes

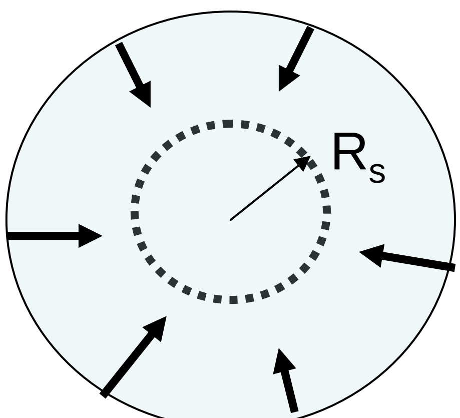


What happens when we squeeze a body of mass M below its Schwarzschild's radius?

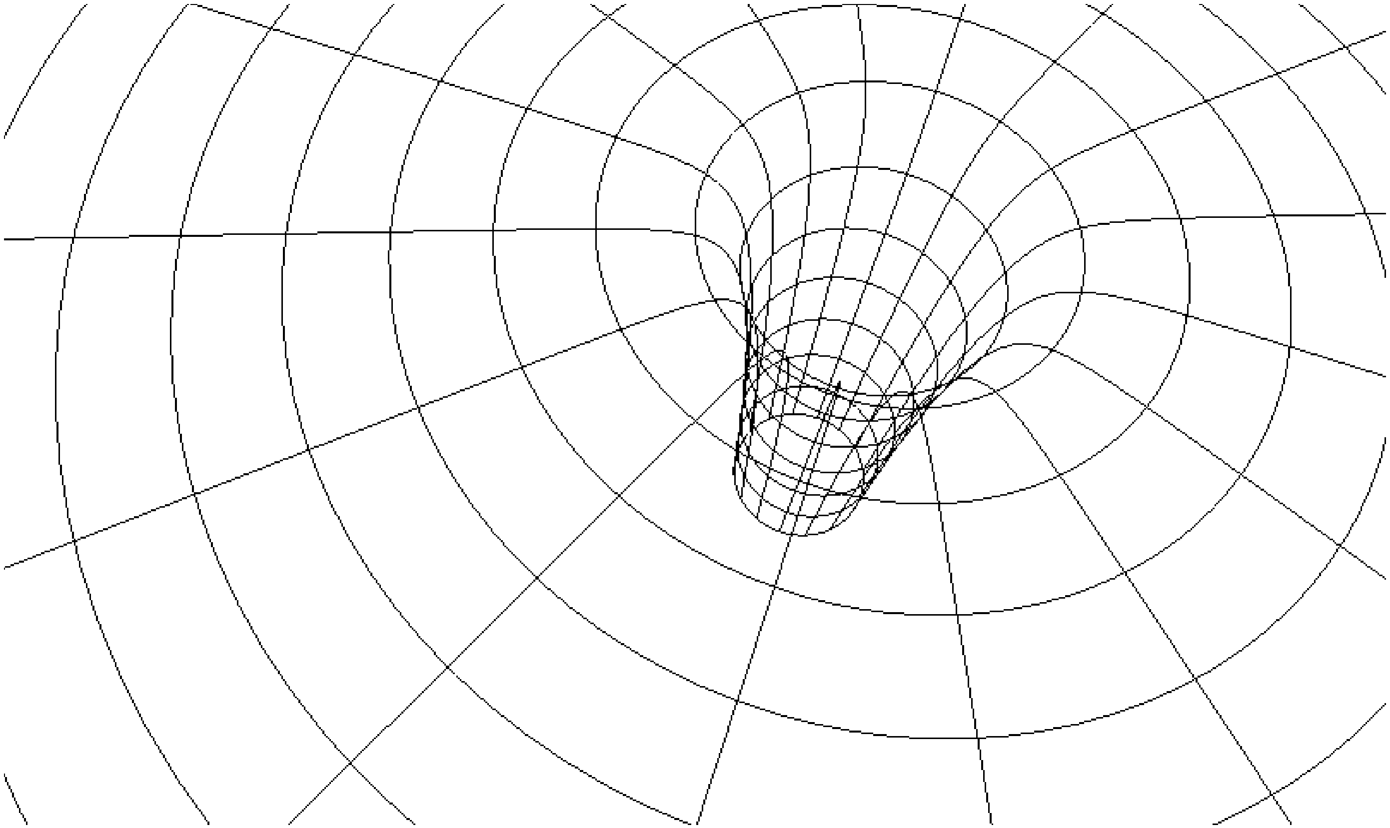
$$R_s = \frac{2GM}{c^2}$$

$$ds^2 = \frac{dr^2}{1 - \frac{R_s}{r}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - \left(1 - \frac{R_s}{r}\right)dt^2$$

K. Schwarzschild 1916

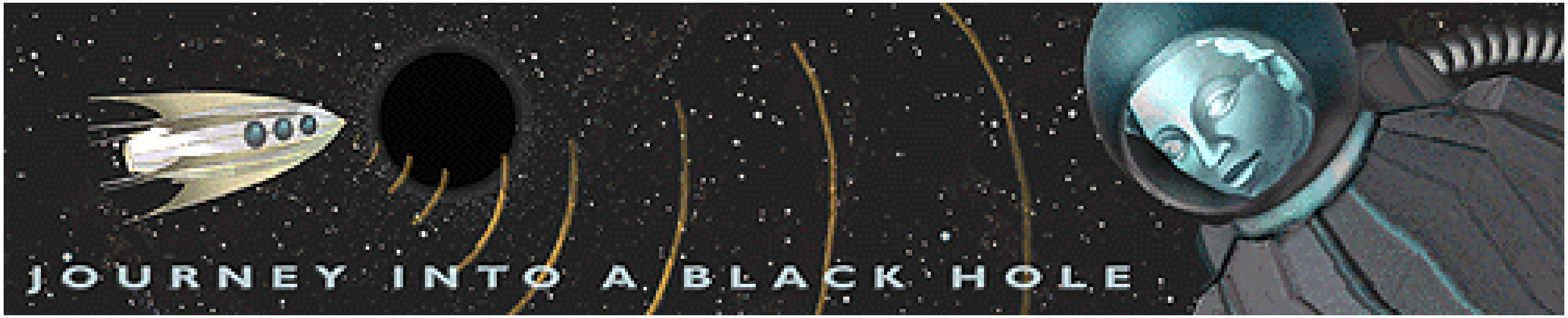


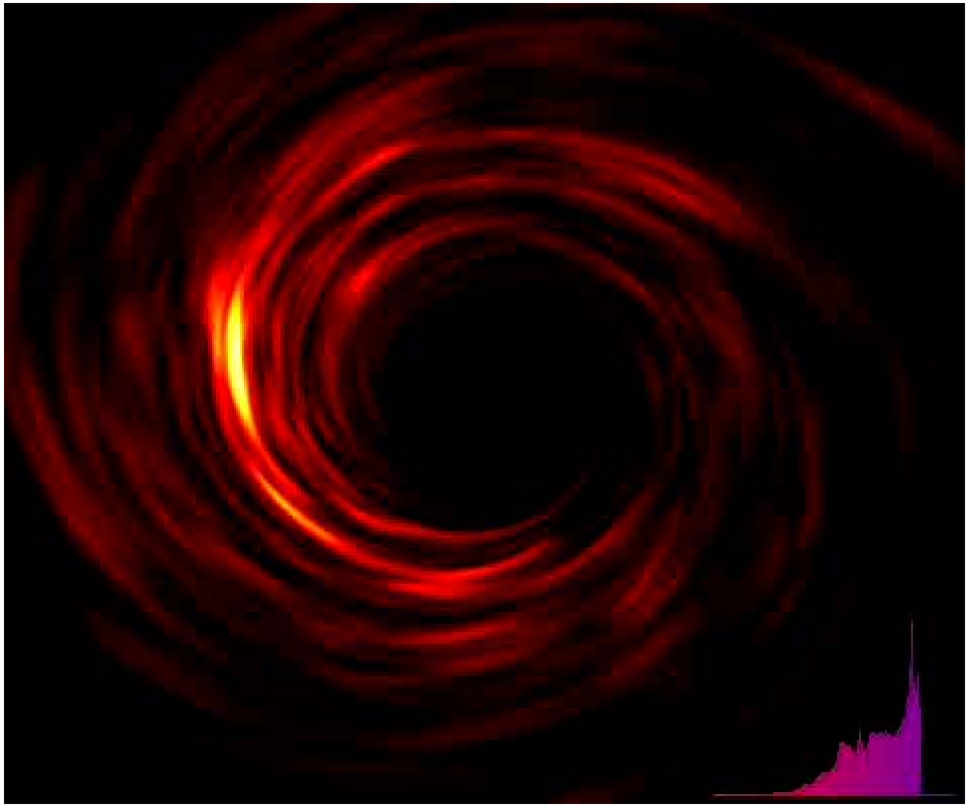
The curvature of a 2D slice of a spherically symmetric black hole



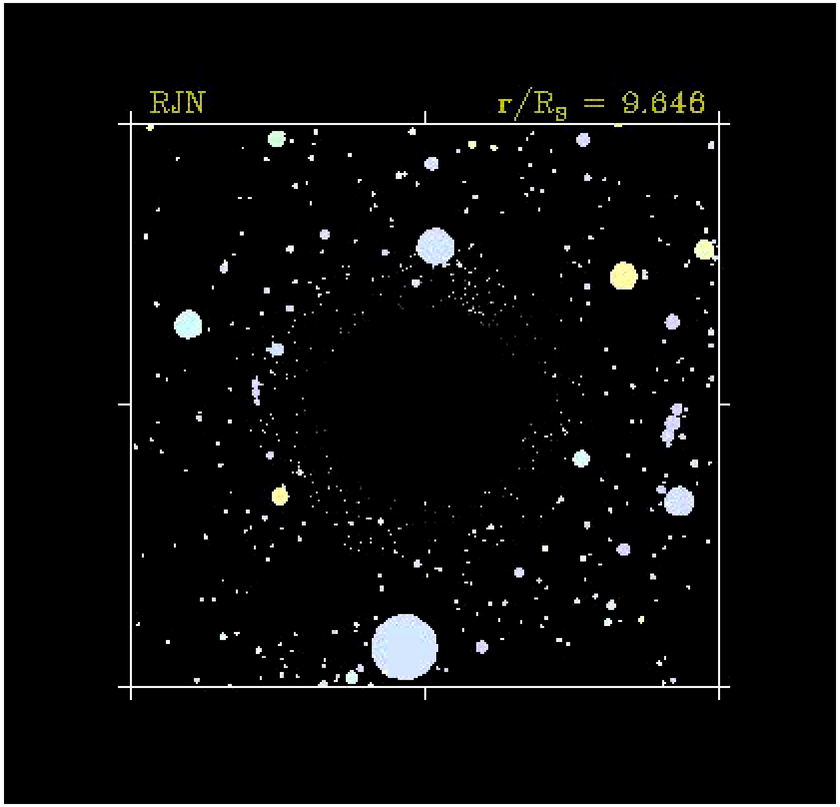
A “well” becomes infinitely deep

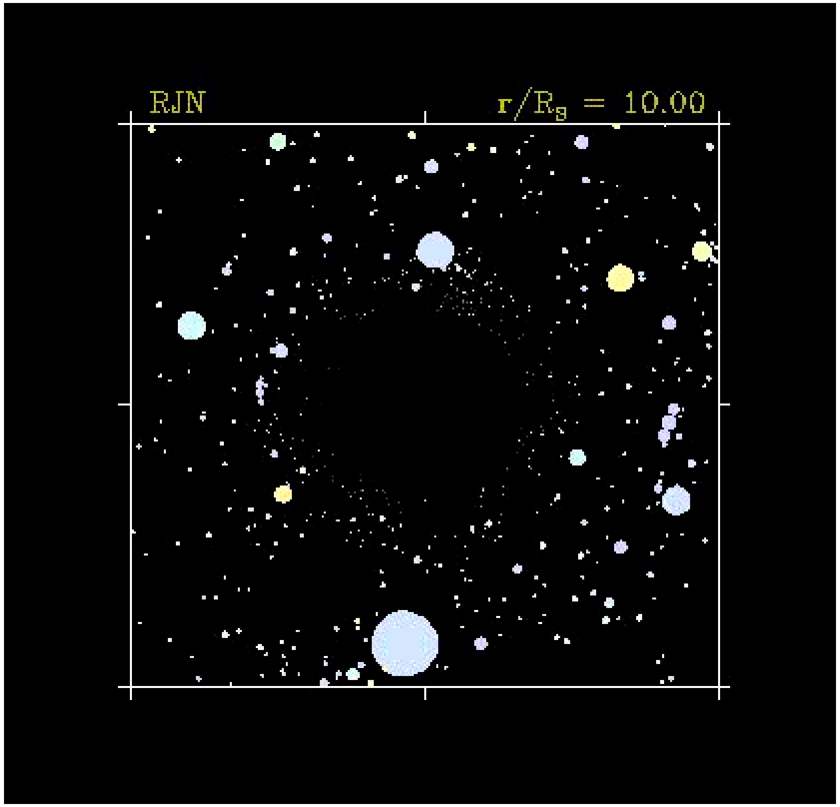
Curvature becomes infinite as we approach the singularity r =0


JOURNEY INTO A BLACK HOLE



Approaching a black hole



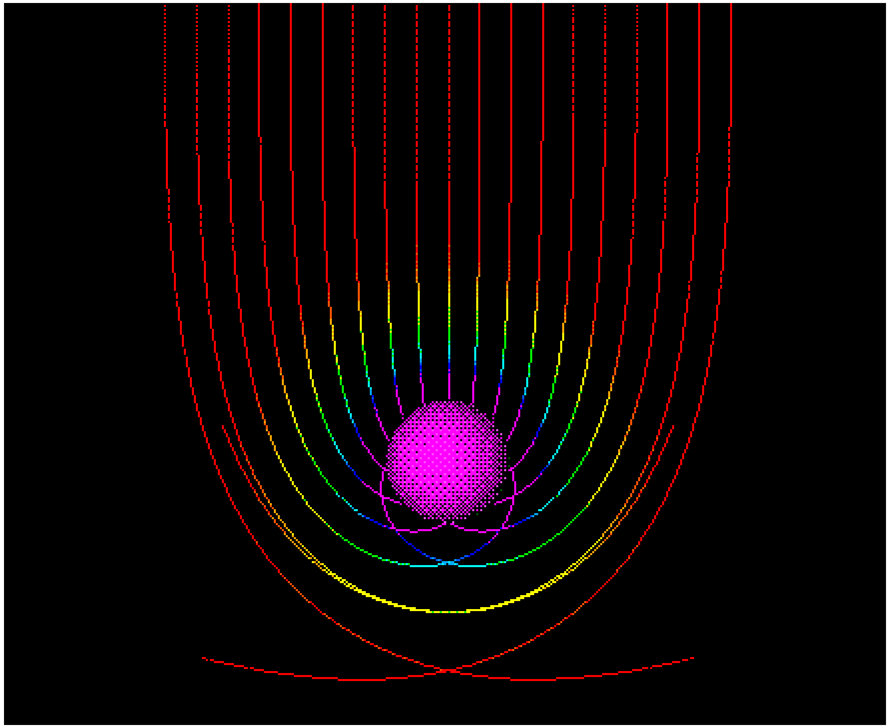


Falling into a black hole

Circling around a black hole

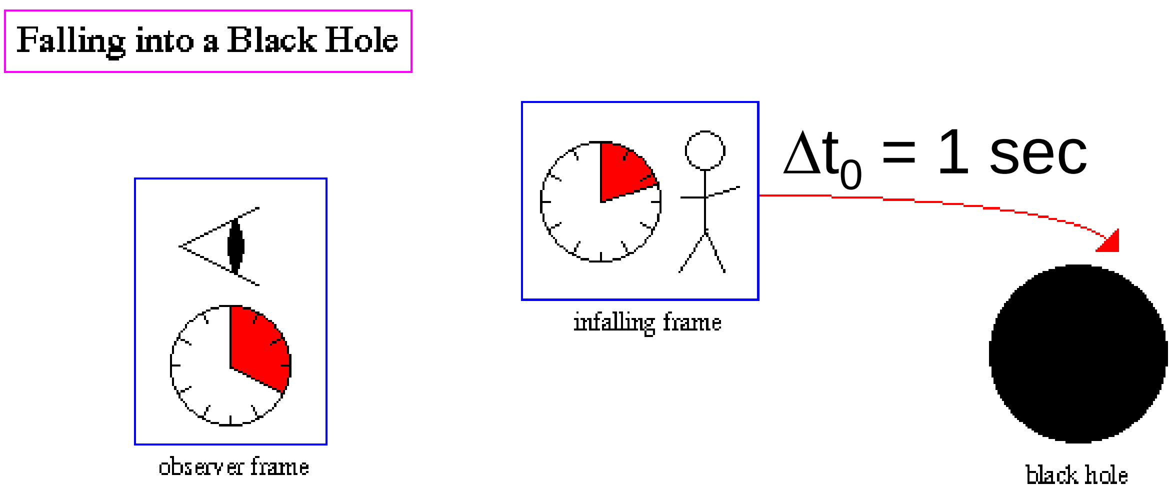
Note the distortion of star images!

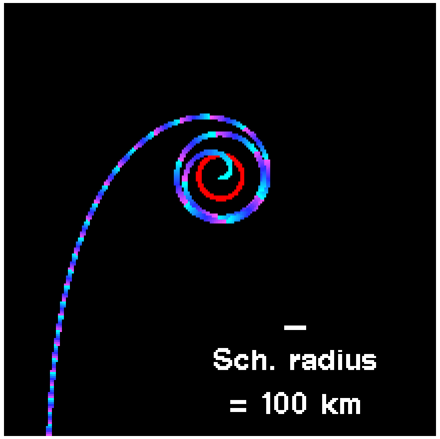
Gravitational bending of light paths around a black hole



Time dilation

$$(\Delta t_0)^2 = -\left(1 - \frac{R_s}{r}\right)(\Delta t)^2 \quad \Rightarrow \quad \Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{R_s}{r}}}$$





$\Delta t > 1$ sec

As measured by a distant observer, clocks slow down when approaching a massive object. Time slows down infinitely when approaching R_s!

$$\text{Frequency } \nu = \frac{1}{\text{Period of oscillations}}$$

⇓

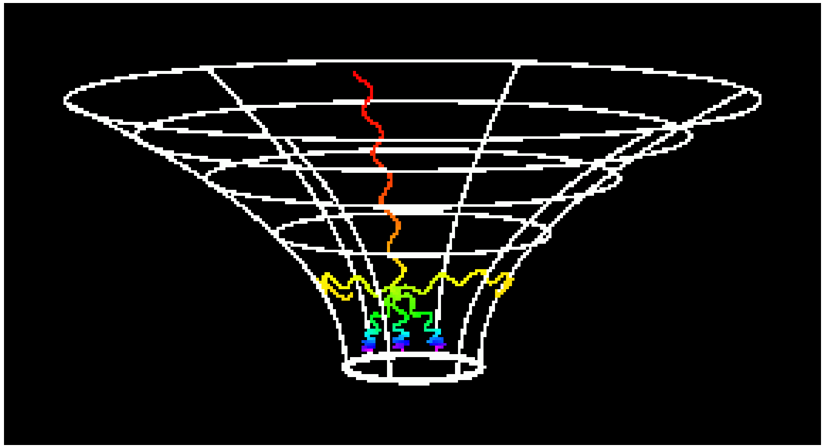
Increase in time intervals means decrease in frequency : Gravitational redshift!

$$f = f_0 \sqrt{1 - \frac{R_s}{r}}$$

Gravitational redshift

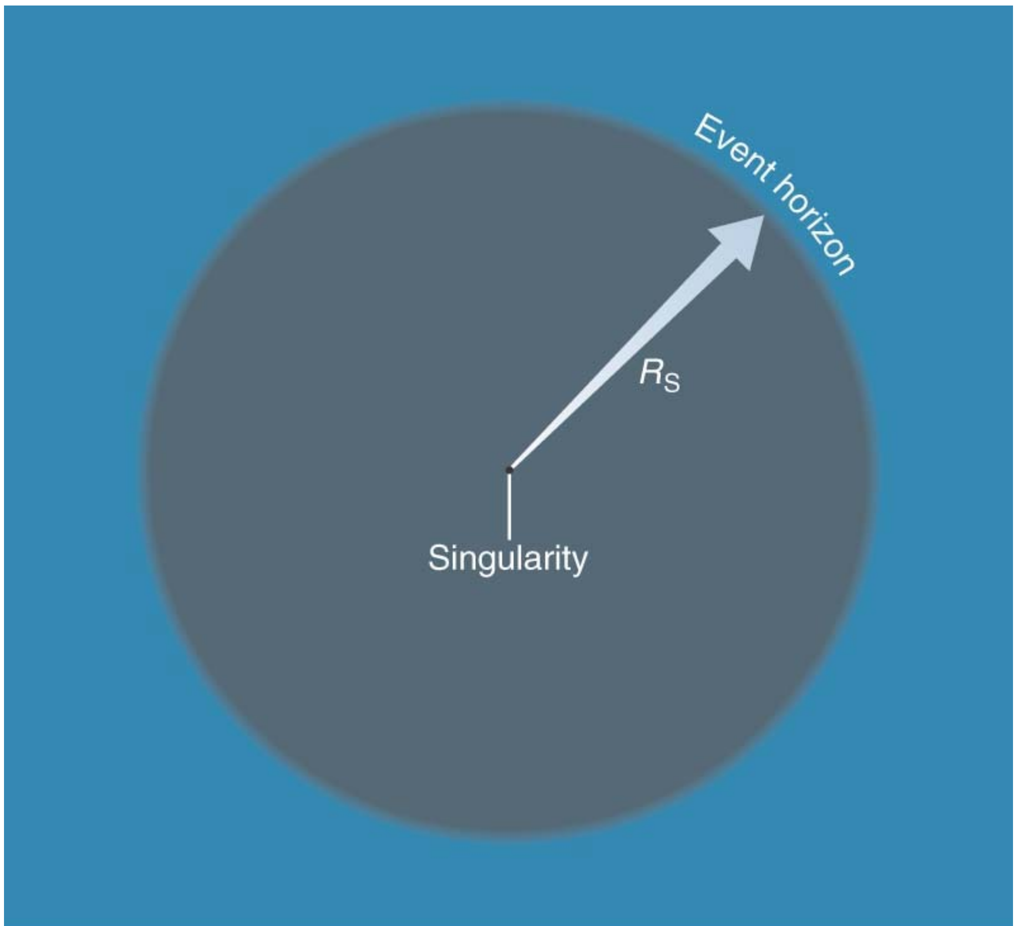
Photons always travel at the speed of light, but they lose energy when traveling out of a gravitational field and appear to be **redder** to an external observer. The stronger the gravitational field, the more energy the photons lose because of this ***gravitational redshift***. The extreme case is a **black hole** where photons from radius R_s lose all their energy.

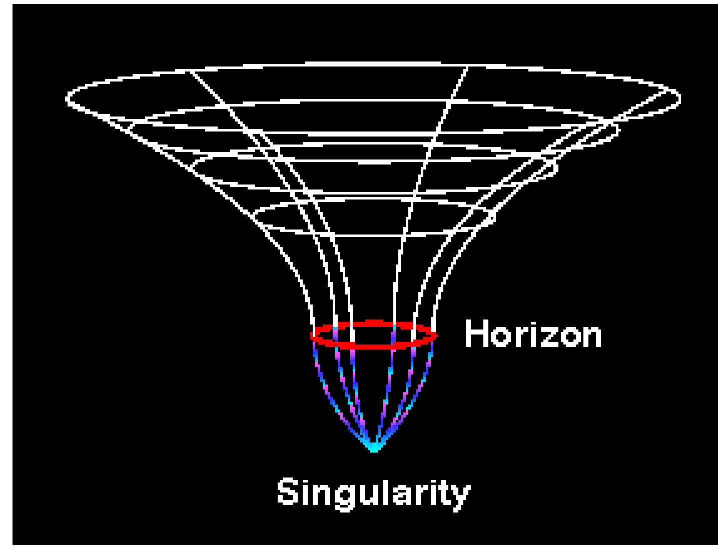
$$f = f_0 \sqrt{1 - \frac{R_s}{r}}$$



Schwarzschild radius: event horizon for a nonrotating body

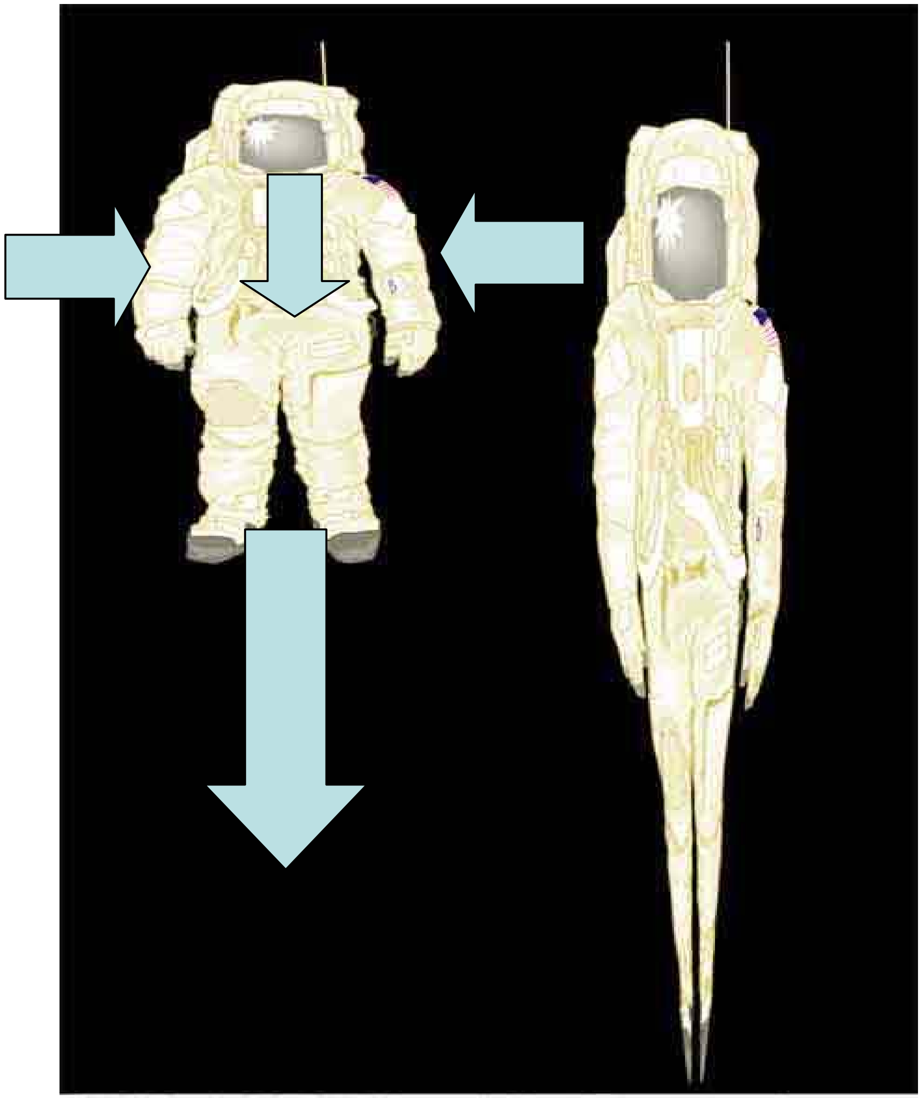
No signals can reach an outside observer from inside the event horizon! This is a point-of-no-return for everything that crosses it.





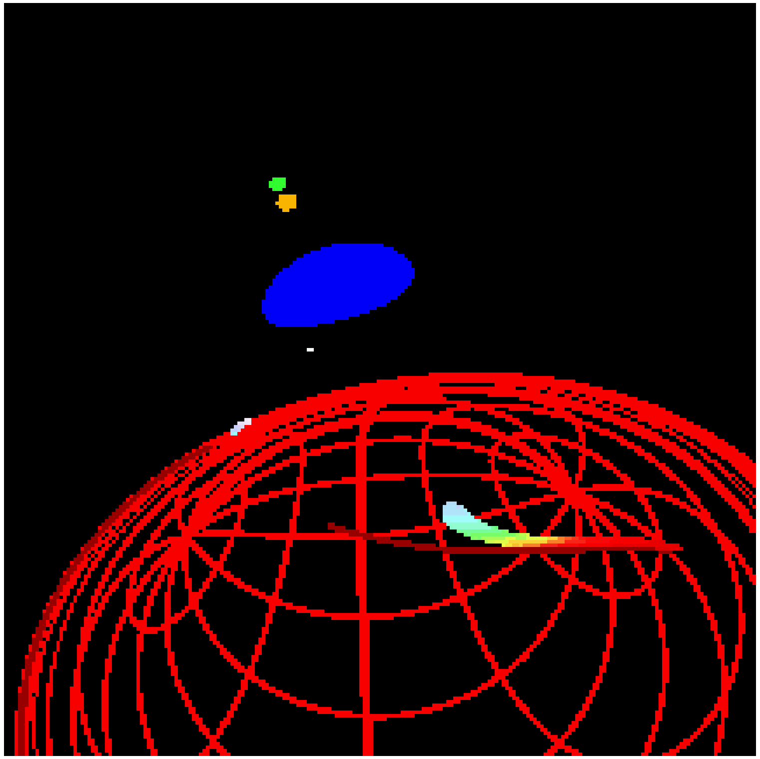
- **No stationary observers below the horizon**
- **You are dragged into a singularity**
- **We experience similar drag due to expansion of space**

Tidal forces and contraction of space squeeze and stretch the astronaut. Lateral pressure is 100 atm at a distance of 100 R_s from the event horizon



- **Longitudinal stretching**
- **Circumferential contraction**

Orbiting a black hole and firing a probe



Diving into a black hole

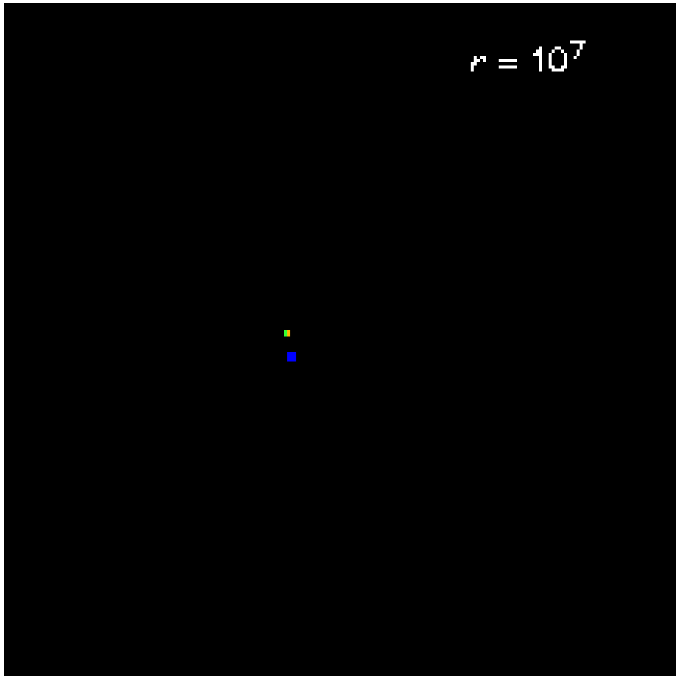


TABLE 11-1
The Schwarzschild Radius

	Mass ($M_\odot$)	R_S
Star	10	30 km
Star	3	9 km
Star	2	6 km
Sun	1	3 km
Earth	0.000003	0.9 cm

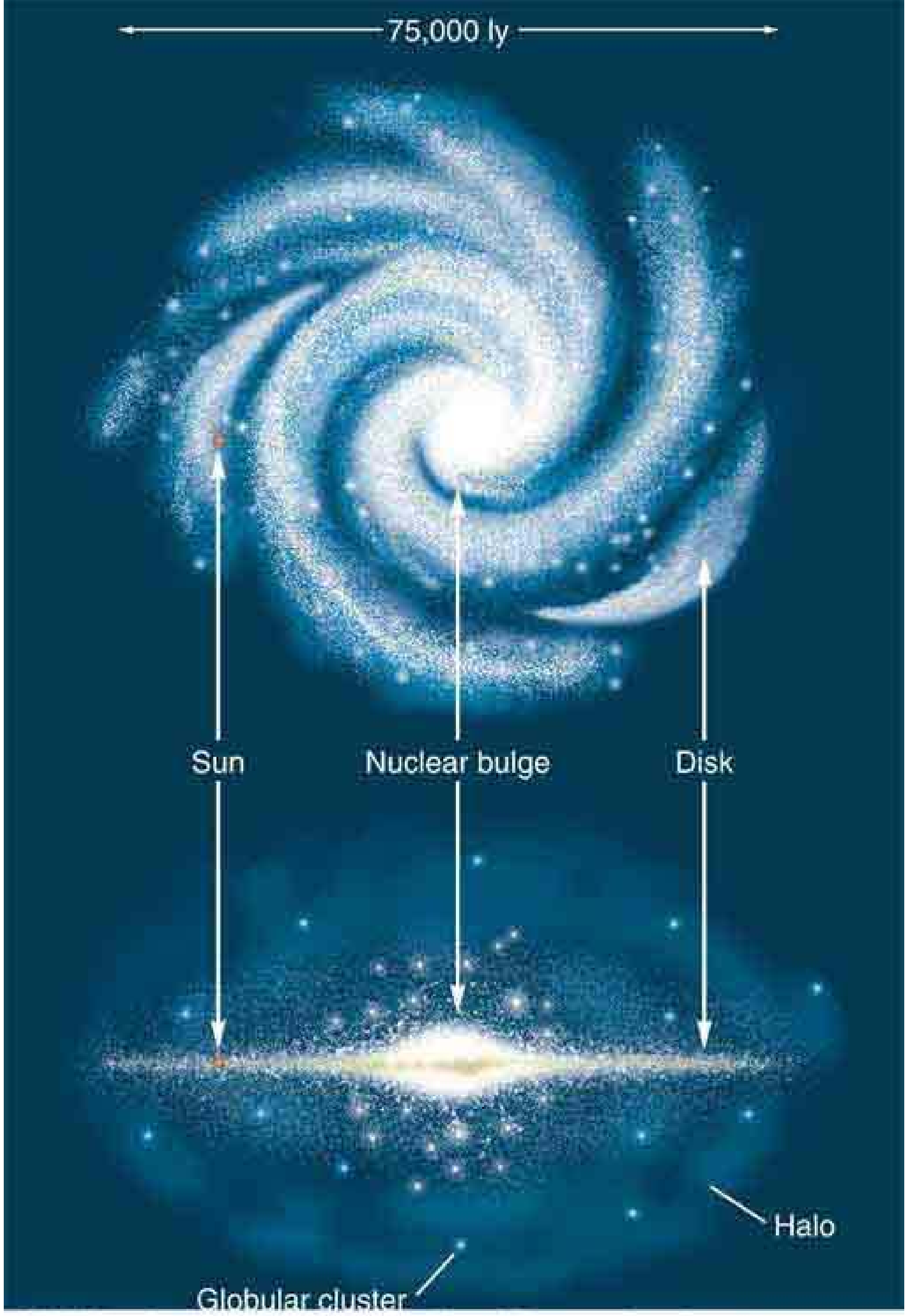
Black holes in the Universe

1. Formation of galaxies
2. Collapse of massive stars
3. Early Universe?

How to find the object that does not emit any radiation?

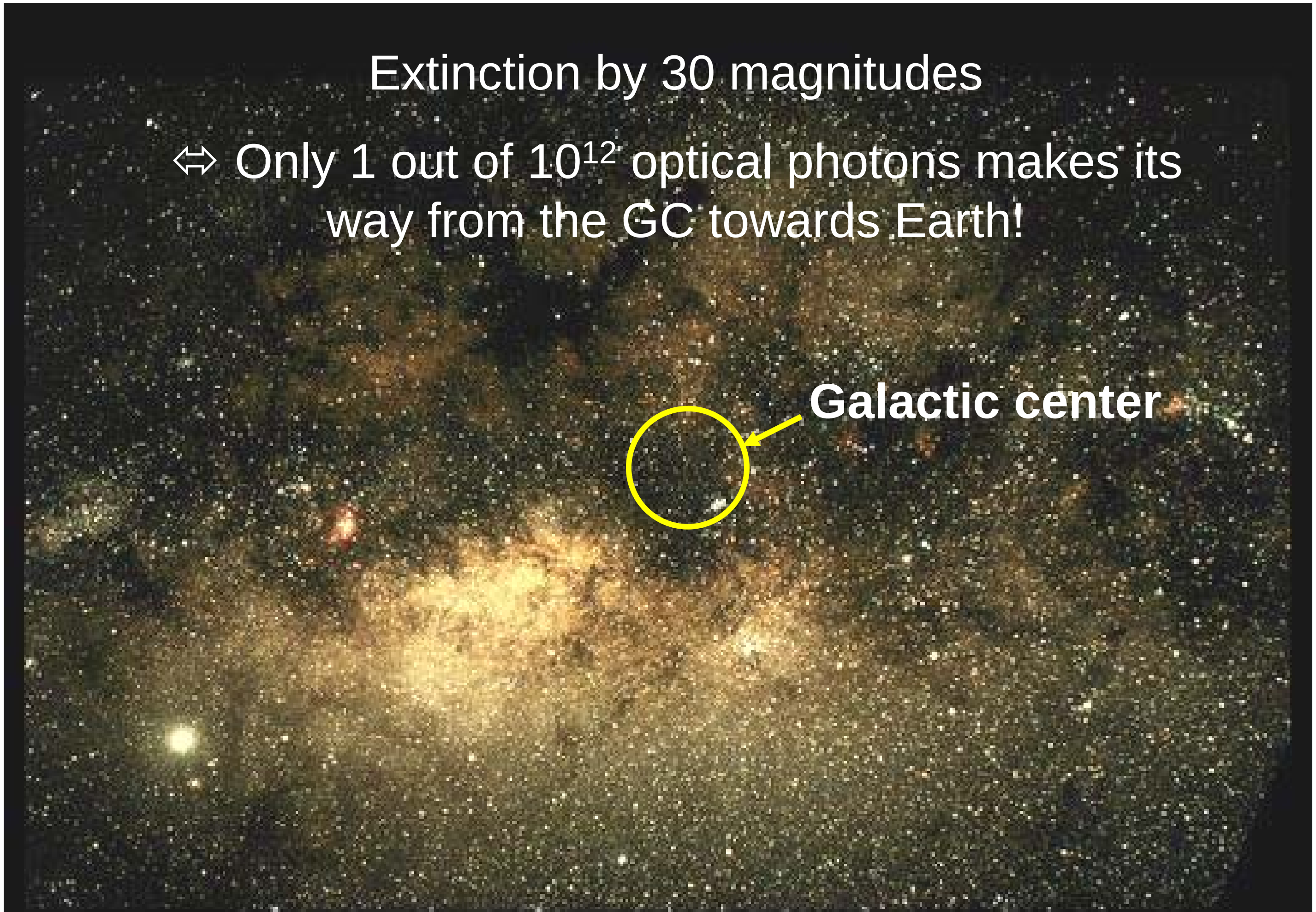
By its effect on nearby objects!

Accretion of surrounding matter onto black holes generates huge amount of heat and radiation

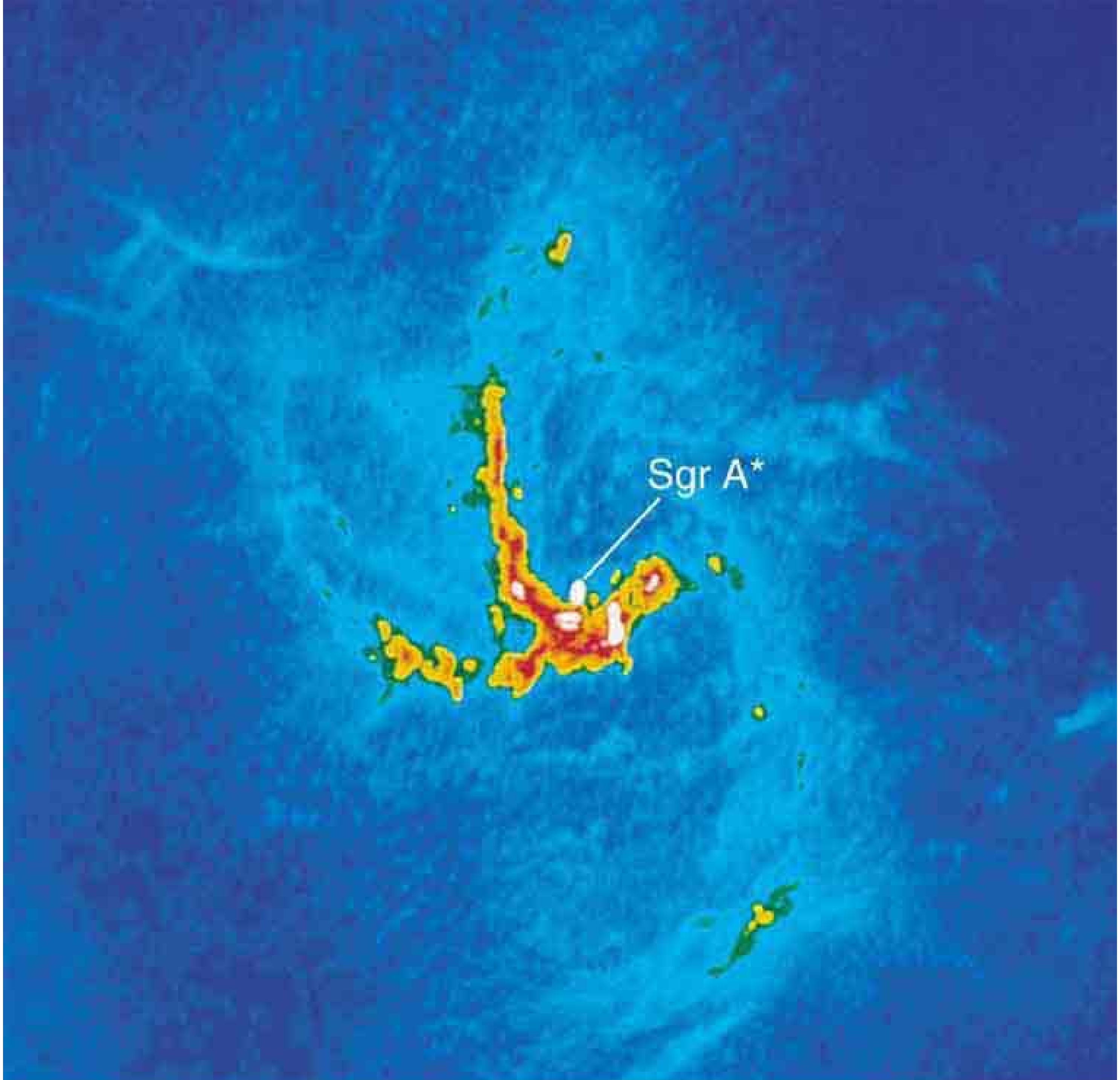

75,000 ly
Sun
Nuclear bulge
Disk
Halo
Globular cluster

The Galactic Center

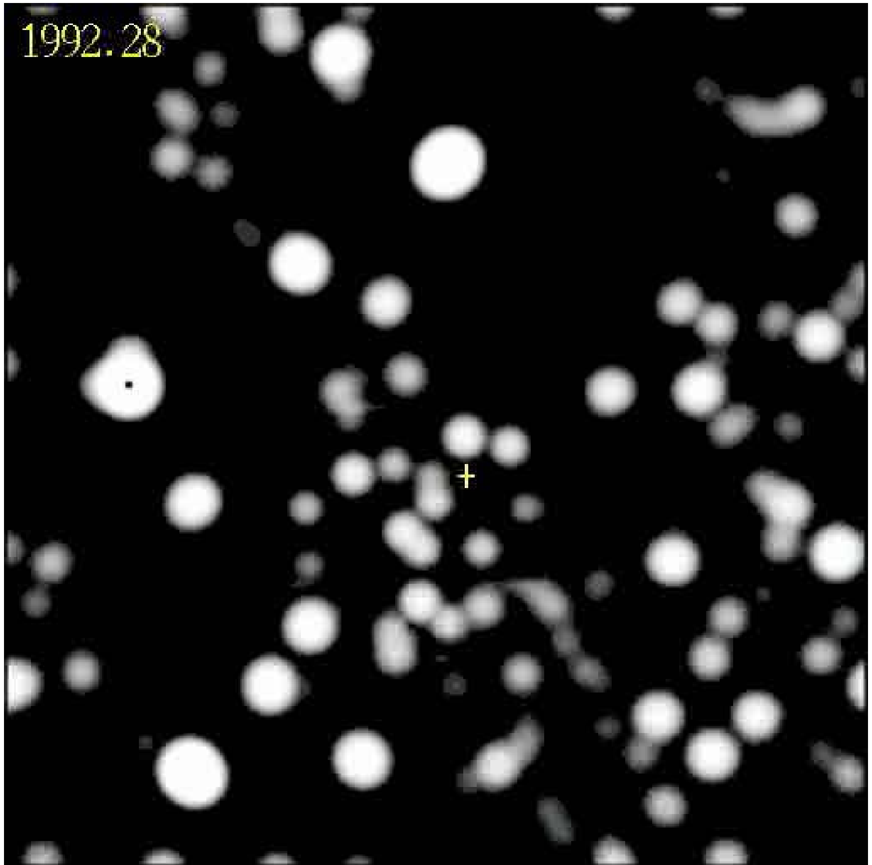
Our view (in visible light) towards the galactic center (GC) is heavily obscured by gas and dust



Wide-angle optical view of the GC region


Sgr A*

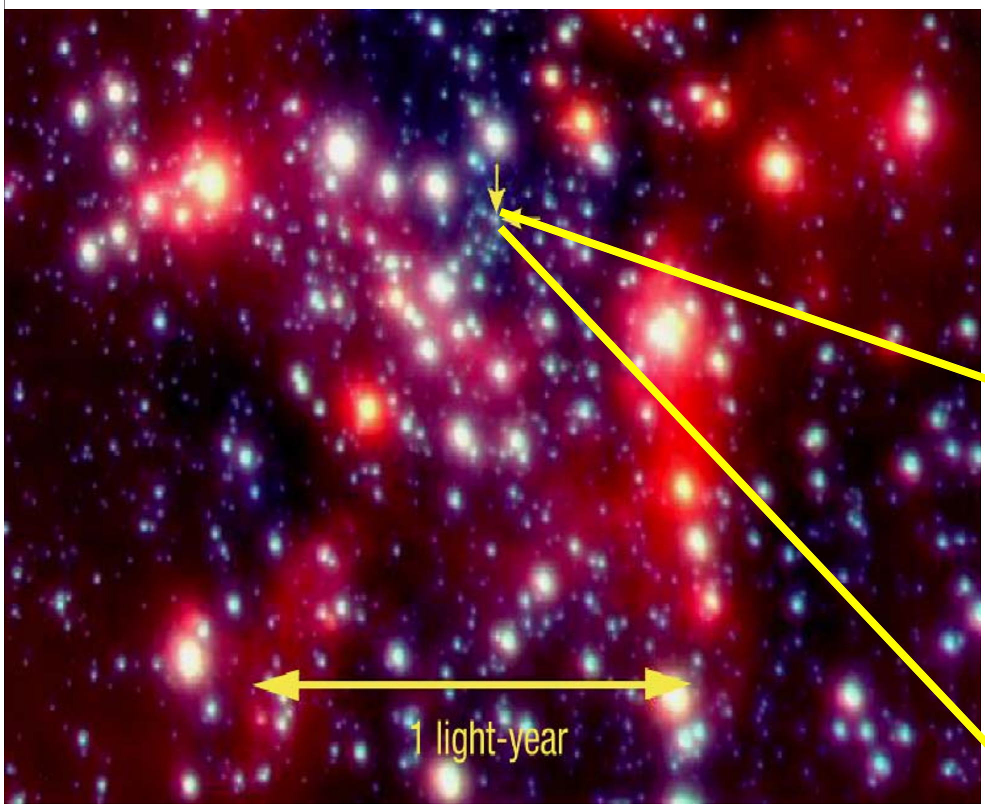
If one looks at this region with big telescopes and near-infrared cameras one can see lots of stars. If one takes pictures every year it seems that some stars are moving very fast (up to 1500 kilometers per second). The fastest stars are in the very center - the position marked by the radio nucleus Sagittarius A* (cross).

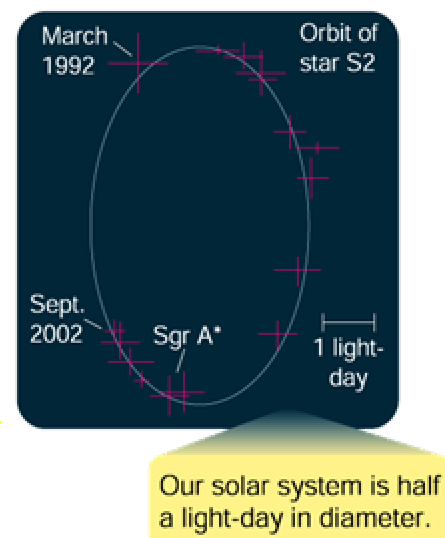


Distance between stars is less that 0.01 pc

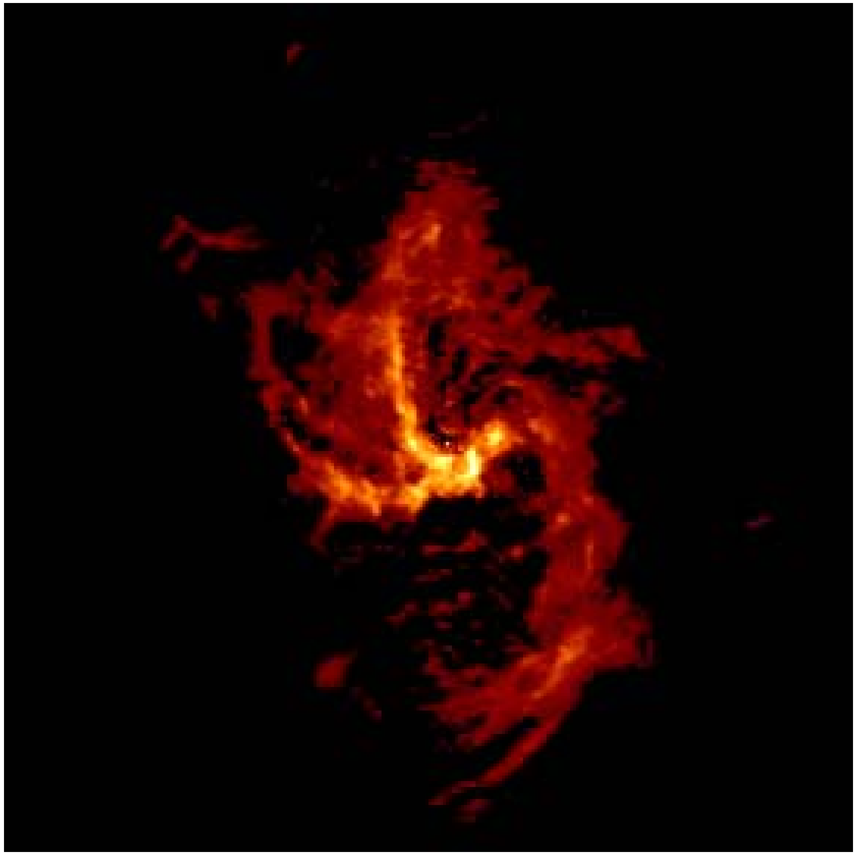
A Black Hole at the Center of Our Galaxy

By following the orbits of individual stars near the center of the Milky Way, the mass of the central black hole could be determined to ~ 2.6 million solar masses

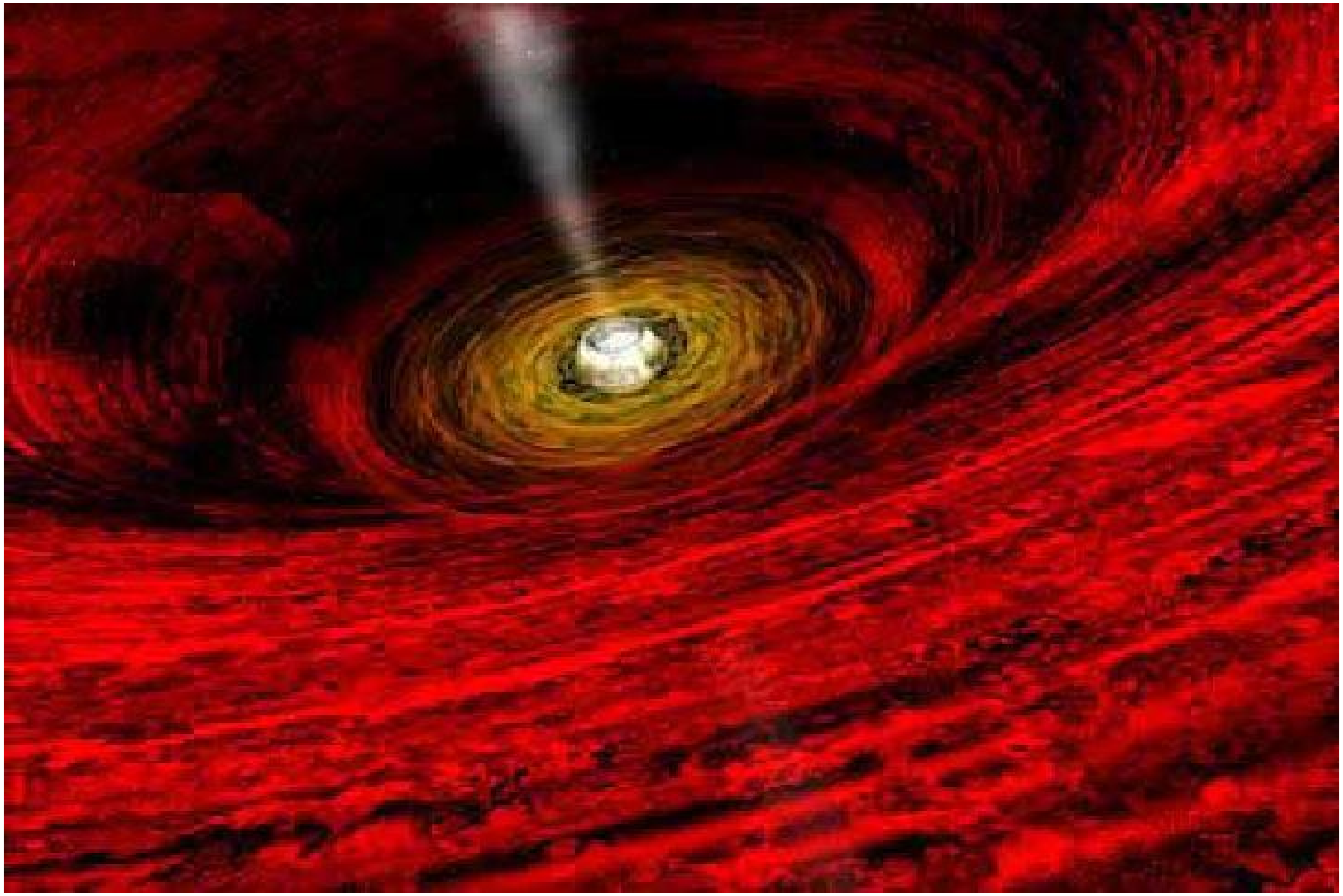




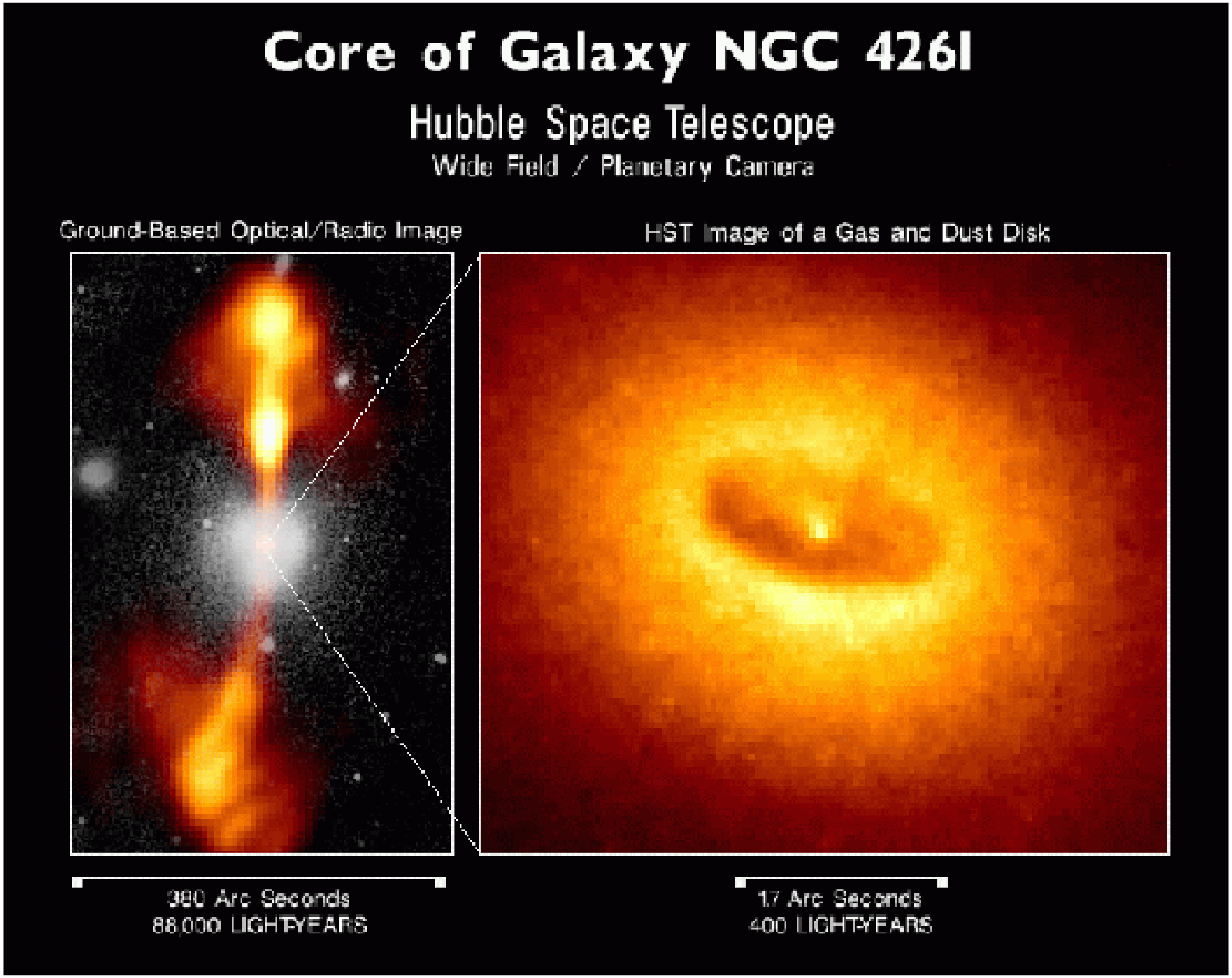
Can we see a “shadow” of a black hole in the Galactic Center??



Black hole vicinity is probably very messy …



Cores of many other galaxies show compact objects in the centers and accretion disks with possible black holes

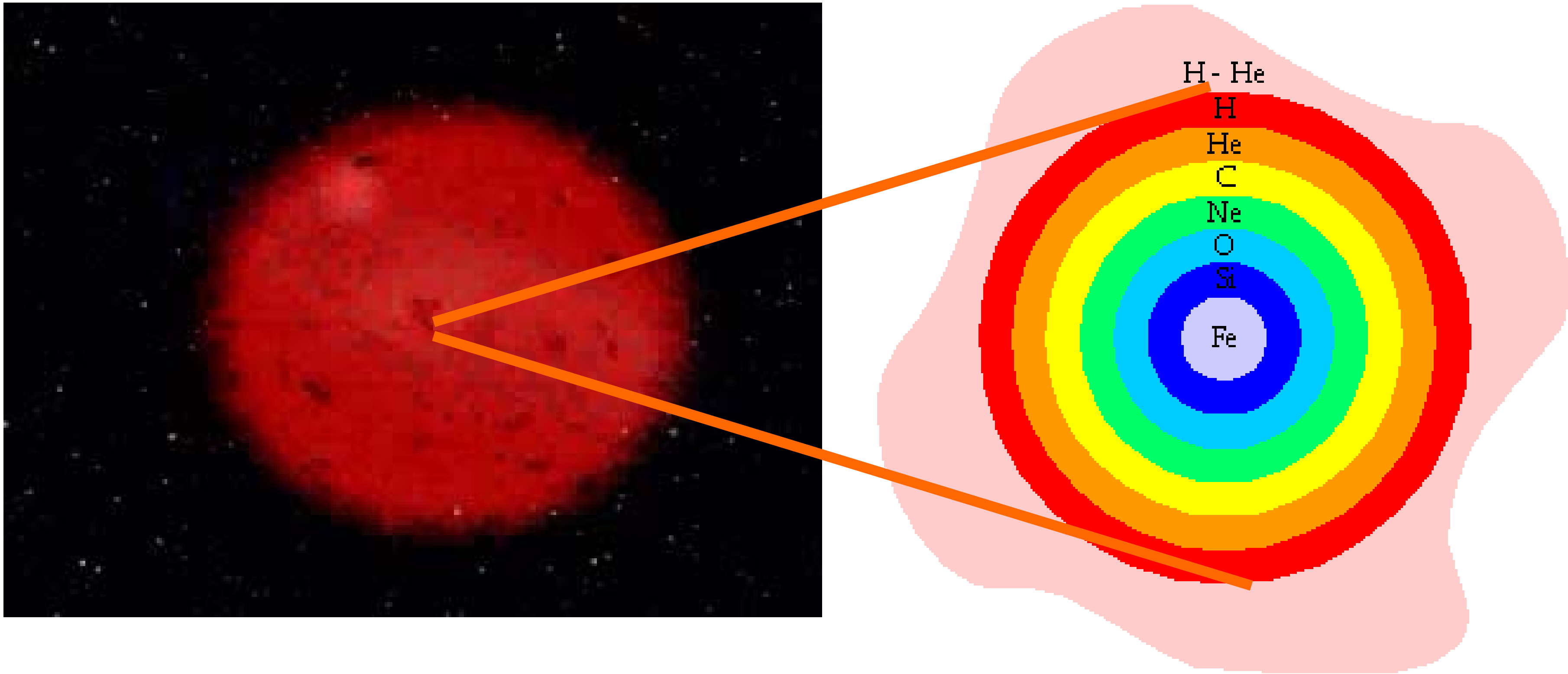


"All hope abandon, ye who enter here"
Dante

Second scenario of black-hole formation:

Death of massive stars

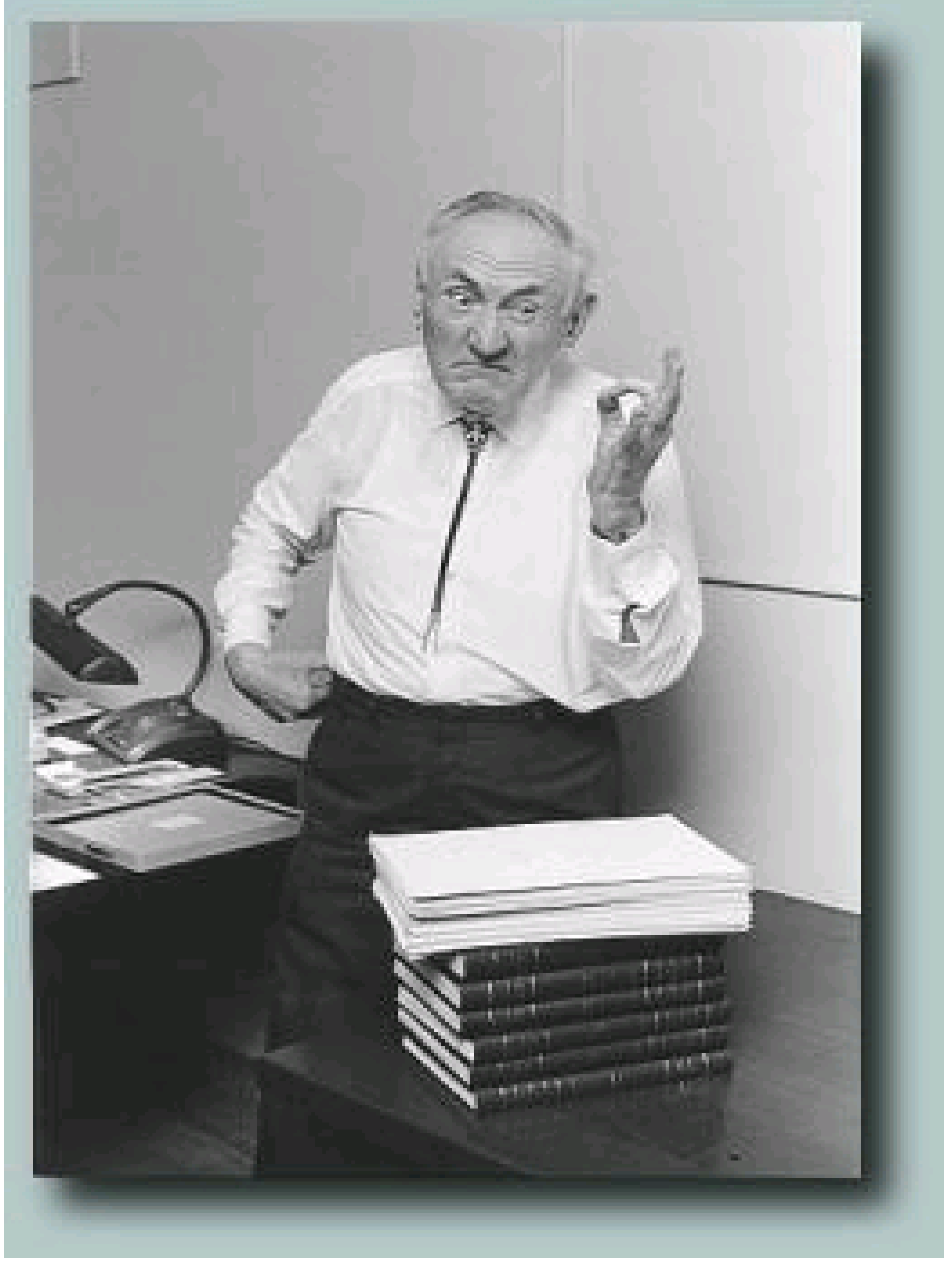
Gravitational collapse of the iron core



Supernova explosion

Fritz Zwicky 1898-1974

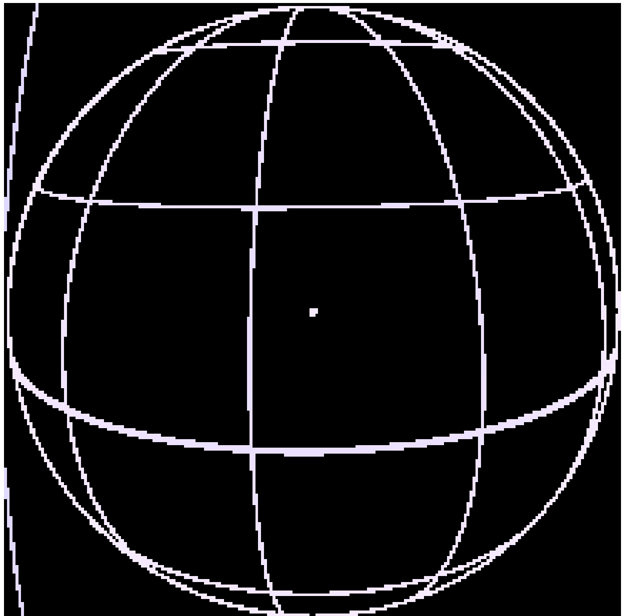
Walter Baade 1893-1960



"spherical bastards”

What would happen *IF* we could observe directly the collapsing stellar core:

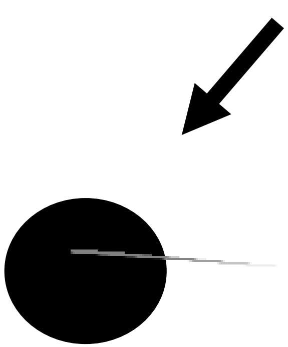
- Photon energies decrease due to a gravitational redshift
- Luminosity decreases due to light bending
- The star becomes dark within a free-fall time of order R/c
- However, from our point of view the collapse slows down to a complete freeze as the star surface approaches the event horizon – time dilation!



Black holes are NOT big cosmic bathtub drains!

Approaching a black hole R ~ R_s (strong field): gravity pull runs away

$$a_g \approx \frac{GM}{R^2\sqrt{1-\frac{R_s}{R}}}$$

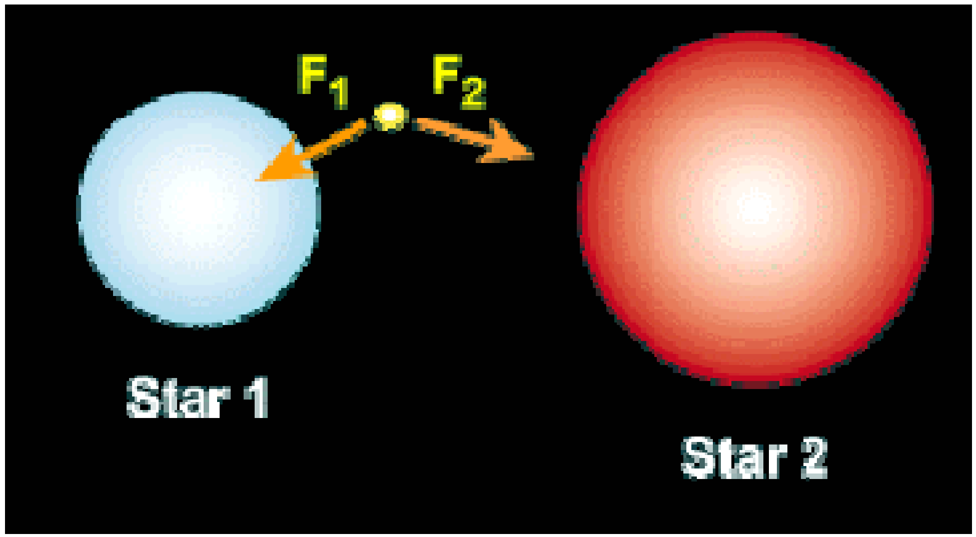


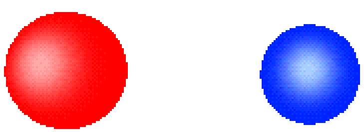
Far from a black hole R >> R_s (weak field): Newtonian gravity law holds

$$a_g \approx \frac{GM}{R^2}$$

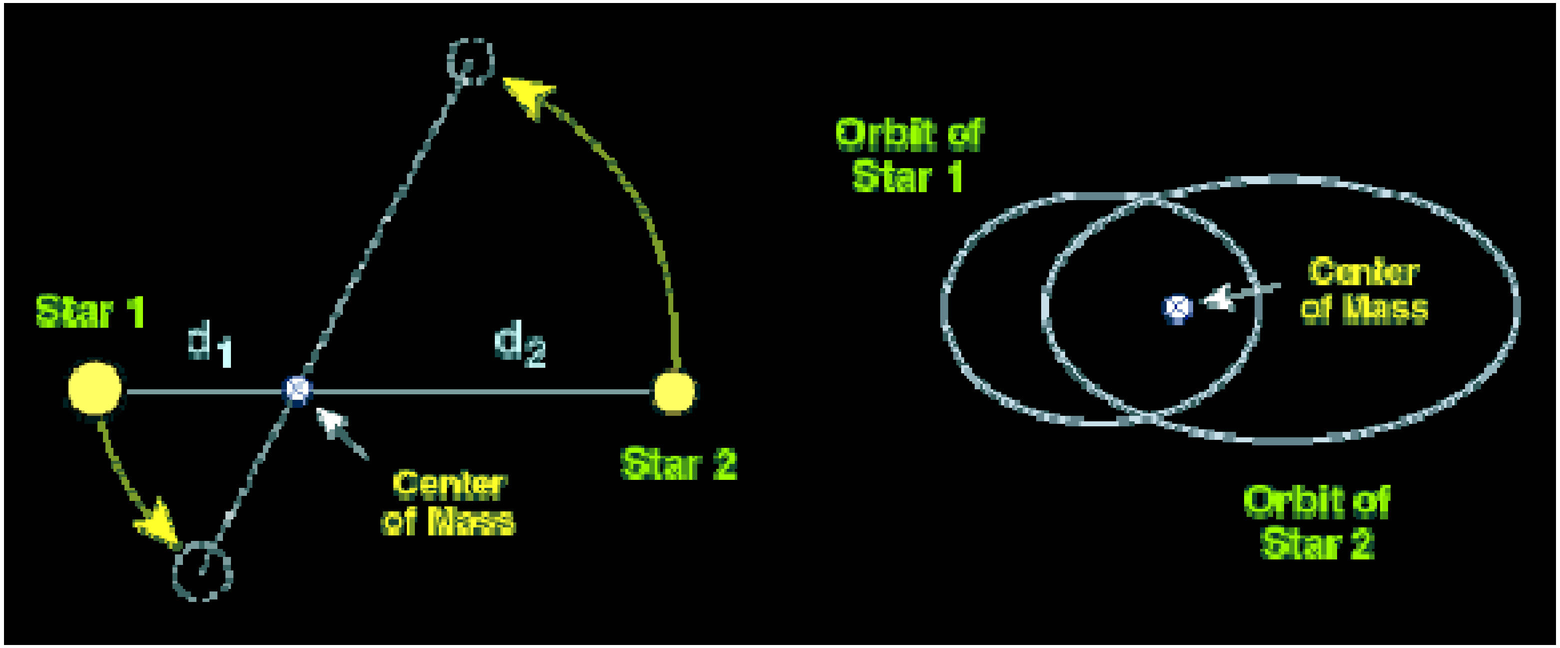
If our Sun collapses into a black hole, we won't see any difference in the gravitational pull (but it will be VERY cold)

Looking for black holes: binary systems





Binary systems



If we can calculate the total mass and measure the mass of a normal star independently, we can find the mass of an unseen companion

$$M_1 + M_2 = \frac{a^3}{P^2};$$

a – in AU
P – in years
M_1+M_2 – in solar masses

Black hole candidates

TABLE 11-2
Nine Black Hole Candidates

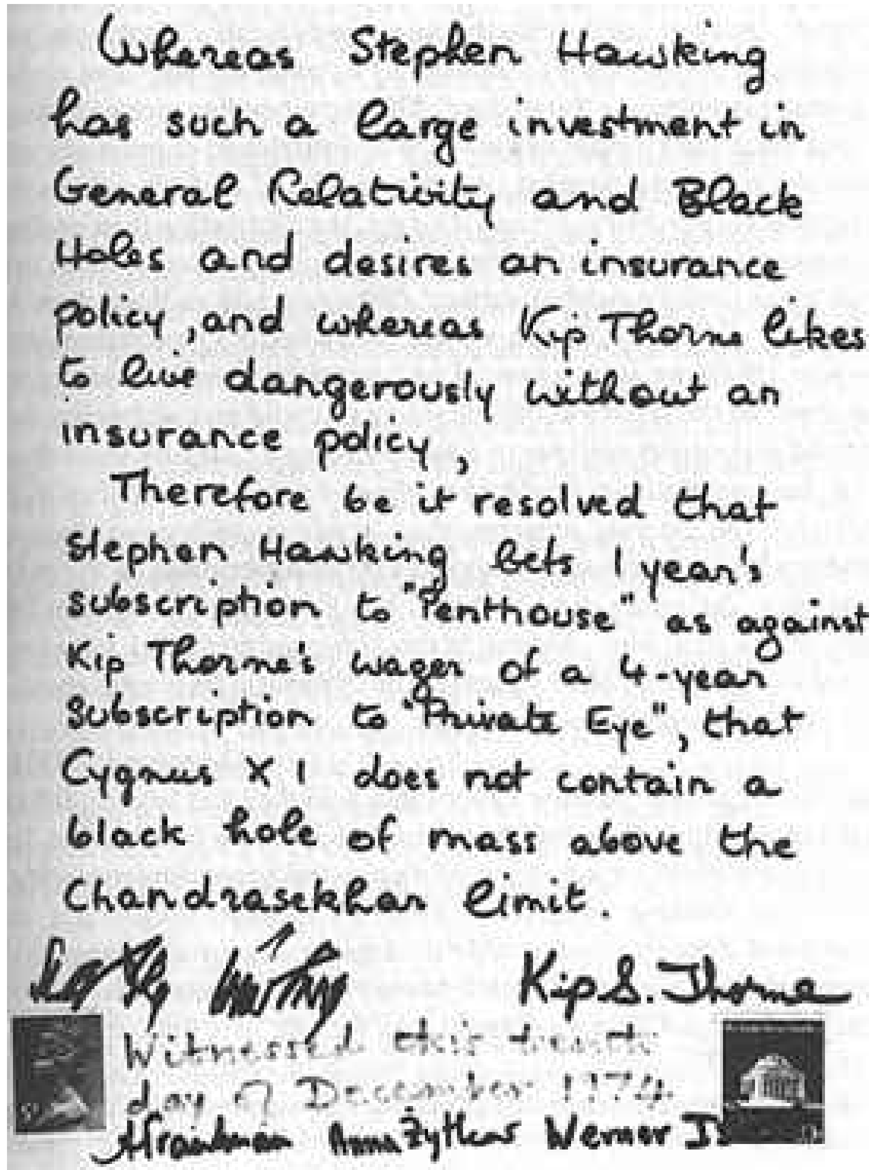
Object	Location	Companion Star	Orbital Period	Mass of Compact Object
Cygnus X-1	Cygnus	O supergiant	5.6 days	>3.8 $M_\odot$
LMC X-3	Dorado	B3 main-sequence	1.7 days	~10 $M_\odot$
V616 Mon	Monocerotis	K main-sequence	7.75 hours	10 ± 5 $M_\odot$
V404 Cygni	Cygnus	K main-sequence	6.47 days	12 ± 2 $M_\odot$
J1655–40	Scorpius	F–G main-sequence	2.61 days	6.9 ± 1 $M_\odot$
QZ Vul	Vulpecula	K main-sequence	8 hours	1 0 ± 4 $M_\odot$
4U 1543-47	Lupus	A main-sequence	1.123 days	2.7–7.5 $M_\odot$
V4641 Sgr	Sagittarius	B supergiant	2.81678 days	8.7–11.7 $M_\odot$
XTEJ1118+480	Ursa Major	K main-sequence	0.170113 days	>6 $M_\odot$

M < 1.4 Solar masses: a white dwarf

M < 3 Solar masses: a neutron star

M > 3 Solar masses: a black hole?

Cygnus X1 – first black hole



Whereas Stephen Hawking has such a large investment in General Relativity and Black Holes and desires an insurance policy, and whereas Kip Thorne likes to live dangerously without an insurance policy,

Therefore be it resolved that Stephen Hawking bets 1 year's subscription to "Penthouse" as against Kip Thorne's wager of a 4-year subscription to "Private Eye", that Cygnus X1 does not contain a black hole of mass above the Chandrasekhar limit.

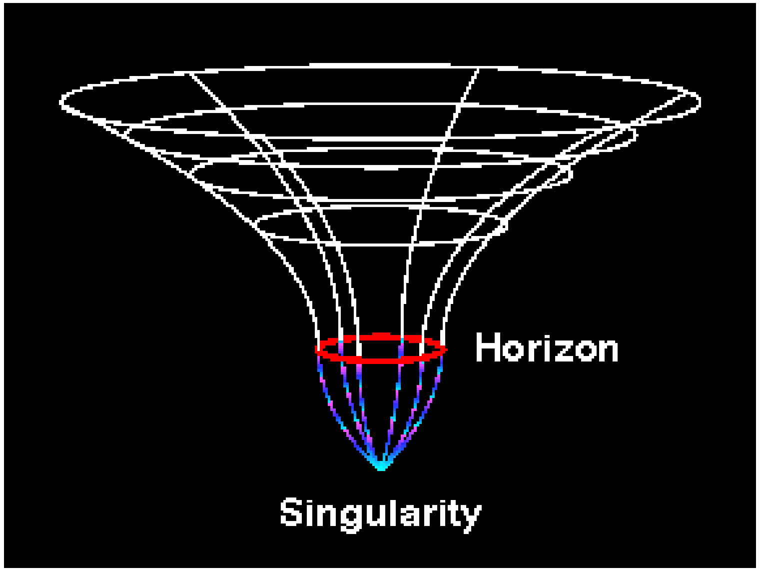
Stephen Hawking Kip S. Thorne

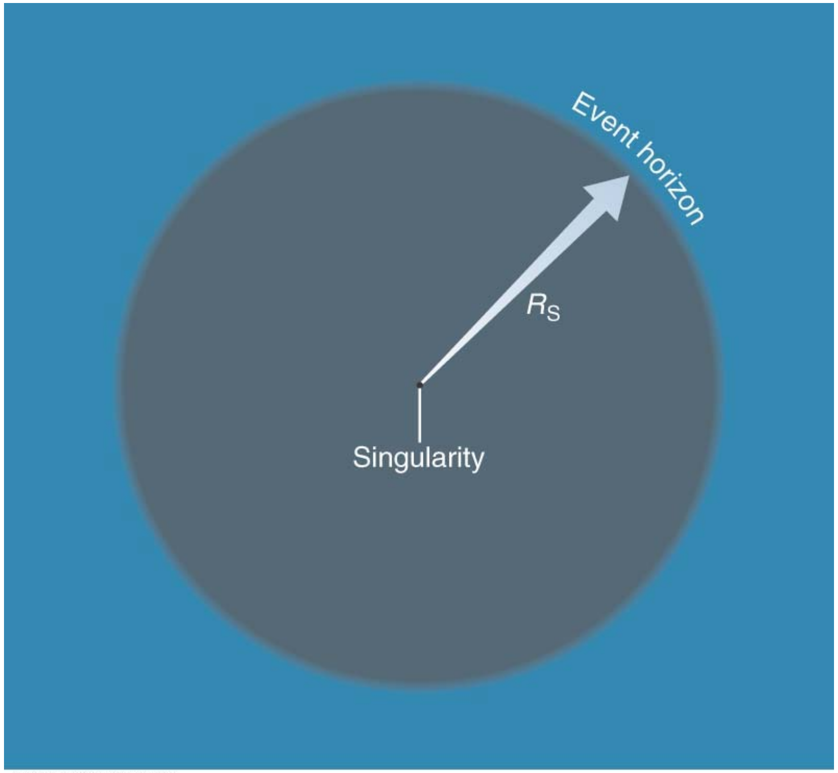
Witnessed this ... day ... December 1974

Anna Zytkow Werner Is...

Some unsolved problems

What is at singularity??





General relativity breaks down at Planck scale

$$l_p = \sqrt{\frac{G\hbar}{c^3}} \sim 1.6\times10^{-35}\ \mathrm{m}$$

Singularities Clothed and Naked

The singularity is the point of infinite density thought to exist at the center of a black hole. We have no way of understanding what would happen in the vicinity of a singularity, since in essence nature divides our equations by zero at such a point. There is an hypothesis, called the "Law of Cosmic Censorship" that all singularities in the Universe are contained inside event horizons and therefore are in principle not observable (because no information about the singularity can make it past the event horizon to the outside world). However, this is an hypothesis, not proven, so it is conceivable that so-called "Naked Singularities" might exist, not clothed by an event horizon.

Whereas Stephen W. Hawking firmly believes that naked singularities are an anathema and should be prohibited by the laws of classical physics,

And whereas John Preskill and Kip Thorne regard naked singularities as quantum gravitational objects that might exist unclothed by horizons, for all the Universe to see,

Therefore Hawking offers, and Preskill/Thorne accept, a wager with odds of 100 pounds stirling to 50 pounds stirling, that when any form of classical matter or field that is incapable of becoming singular in flat spacetime is coupled to general relativity via the classical Einstein equations, the result can never be a naked singularity.

The loser will reward the winner with clothing to cover the winner's nakedness. The clothing is to be embroidered with a suitable concessionary message.

John P. Preskill Kip S. Thorne

Stephen W. Hawking John P. Preskill & Kip S. Thorne
Pasadena, California, 24 September 1991

Conceded on a techicality; 5 Feb. 1997 Stephen W. Hawking

Some black hole and GR pages used in this lecture:

http://www.damtp.cam.ac.uk/user/gr/public/bh_home.html

http://www.star.le.ac.uk/~sav2/blackholes/news.html

http://www.faculty.iu-bremen.de/course/fall02/GeneralGeoAstro1/students/BlackHoles/Black%20holes%20and%20Schwartzschild%20geometry.htm

http://casa.colorado.edu/~ajsh/dive.html

http://archive.ncsa.uiuc.edu/Cyberia/NumRel/BlackHoleAnat.html

http://www.astro.ku.dk/~cramer/RelViz/

http://www.phys.lsu.edu/astro/movie_captions/motl.binary.html

http://www.phy.syr.edu/courses/modules/LIGHTCONE/index.html

http://www.ukaff.ac.uk/movies.shtml

http://csep10.phys.utk.edu/guidry/violence/index.html

http://cassfos02.ucsd.edu/public/astroed.html