Chapter 7

Last time:

$\vec{A} \cdot \vec{B}$ dot product

$= |\vec{A}| |\vec{B}| \cos\theta$

$= A_x B_x + A_y B_y$

Work:

$$W^{\vec{F}}_{\vec{r}_1, \vec{r}_2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

$$= \int_{\vec{r}_1}^{\vec{r}_2} \overbrace{(F_x \vec{i} + F_y \vec{j})}^{\vec{F}} \cdot \overbrace{(dx\vec{i} + dy\vec{j})}^{d\vec{r}}$$

$$\boxed{W^{\vec{F}}_{\vec{r}_1, \vec{r}_2} = \int_{x_1}^{x_2} F_x\, dx + \int_{y_1}^{y_2} F_y\, dy}$$

Kinetic Energy

$K.E. = 1/2\, m v^2$

Work-Energy Theorem

$$W^{total} = KE_f - KE_i$$

$$= \boxed{1/2\, m v_f^2 - 1/2\, m v_i^2}$$

$W^{total} > 0 \Rightarrow 1/2\, m v_f^2 > 1/2\, m v_i^2$

$W^{total} < 0 \rightarrow 1/2\, m v_f^2 < \frac{1}{2} m v_i^2$

- Examples
- Spring Force (variable force)
- Power
- Quiz

Example:

Drop object of mass m from a height H. Calculate the final velocity of the object.

$$\left[y = y_0 + v_0 t + \tfrac{1}{2} a t^2 \quad \text{"old way"} \right] \quad a = -g$$

Now use Work–Energy Theorem

$$W^{total} = KE_f - KE_i$$

$\downarrow F_g = -mg$

$$W^{total} = \underline{W^{F_g}_{y=H,0}}$$

$$W^{F_g}_{y=H,0} = \cancel{\int_0^0 F_x\,dx}^{\,0} + \int_{H=y_i}^{0=y_f} F_y\,dy$$

$$= \int_H^0 (-\underline{mg})\,dy$$

$$= -mgy \Big|_H^0 = \left[0 - (-mgH) \right]$$

$$\boxed{W^{F_g}_{y=H,0} = \underline{mgH}}$$

$$W^{total} = \frac{1}{2} m v_f^2 - \cancel{\frac{1}{2} m v_i^2}^{\,0}$$

$$\cancel{m} g H = \frac{1}{2} \cancel{m} v_f^2$$

$$v_f = \sqrt{2gH}$$

$$\vec{v}_f = -\sqrt{2gH}\,\vec{j}$$

$$W^{\boxed{F_g}}_{\underbrace{y=h,0}} = mgh \quad \downarrow F_g \quad \downarrow \vec{y} = y_f - y_i$$

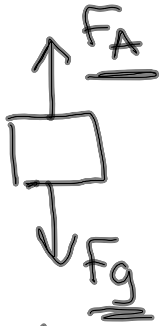
object is lowered (goes down)

$$W^{F_g}_{y=0,h} = -mgh \quad \uparrow \vec{y} \quad \downarrow \vec{F}_g$$

$y_f - y_i$

Problem #1

1. $m = 100\,kg$
lowered 2m at
constant velocity



$a = 0$

$\sum F = ma = 0$ → "old way"

Now use Work–Energy Theorem
to calculate F_A.

a.) How much work is done
by gravity?

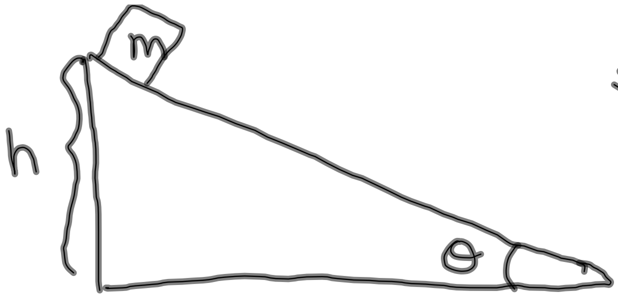
$$W^{F_g}_{y=2,0} = mgh = (100kg)(9.8\tfrac{m}{s^2})(2m)$$
$$= \boxed{1960\,J}$$

$$W^{total} = W^{F_g} + W^{F_A} = KE_f - KE_i$$
$$= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$
$$v_i = v_f$$
$$= 0$$

$$W^{F_g} + W^{F_A} = 0$$

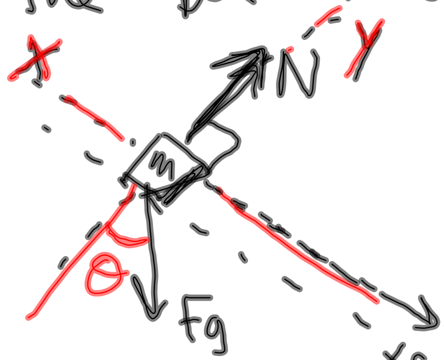
b.) What is the magnitude of F_A?

$$W^{F_A}_{y=2,0} = \int_2^0 F_A\,dy = F_A \int_2^0 dy$$
$$= F_A y \Big|_2^0$$
$$= F_A(-2) = -1960\,J$$
$$\boxed{F_A = 980\,N}$$



starts from rest at the top

Calculate the velocity at the bottom of the incline.



$$\underline{W^{total}} = \underline{\tfrac{1}{2} m v_f^2} - \cancel{\tfrac{1}{2} m v_i^2}^{\,0}$$

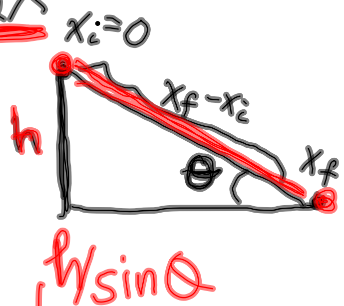
$$W^N = 0 = \int_{x_i}^{x_f} \cancel{N_x}^{\,0}\, dx + \int_0^0 N_y\, dy$$

$$W^{F_g} = mgh = \int \underline{F_{gx}}\, dx + \int_0^0 F_{gy}\, dy$$

$$= \int_0^{\frac{h}{\sin\theta}} \underline{mg\sin\theta}\, \underline{dx}$$

$$\sin\theta = \frac{h}{x_f}$$

$$x_f = \frac{h}{\sin\theta}$$



$$= mg\sin\theta \cdot x \Big|_0^{h/\sin\theta}$$

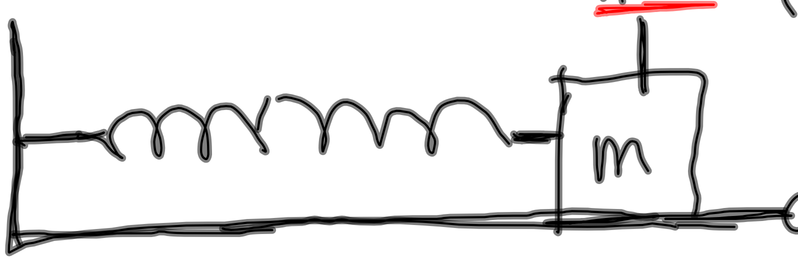
$$= mg\sin\theta \cdot \frac{h}{\sin\theta} - 0$$

$$= mgh$$

Spring Force (Physical Example of a varying force)

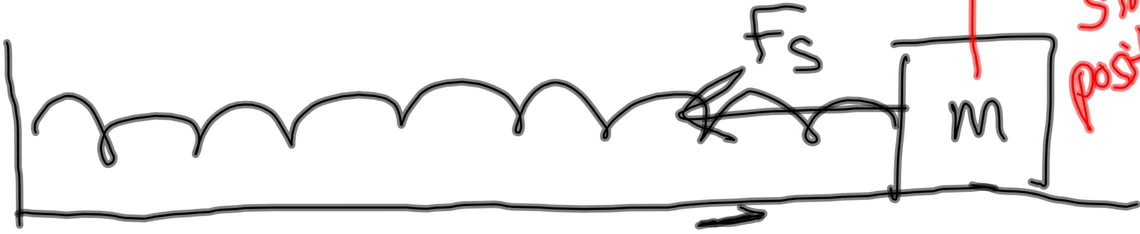
Consider an "ideal" spring

$x=0$ ("equilibrium position" unstretched and uncompressed)



$$\sum F_x = F_s = 0$$

$x>0$ stretched position

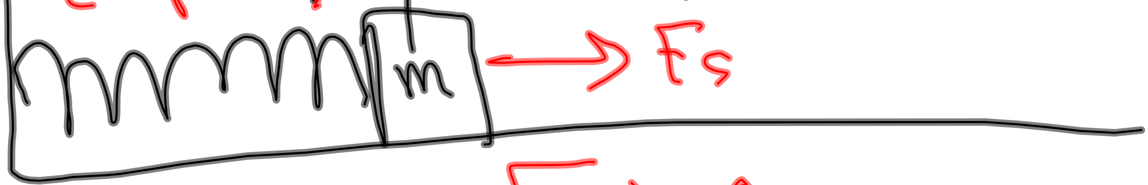


$$\vec{F}_s < 0$$

$$F_s = -Kx$$

K "spring constant"

compressed $x<0$



$$F_s > 0$$

$$F_s = -K\underline{x}$$ (direction is opposite of displacement)

Work done by Spring force

Horizontal force (only along x and motion is along x)

Starts at $x=A$ & moves to $x=0$

What is work done by F_s in this motion?

$$W^{F_s}_{x=A,0} = \int_A^0 F_x\,dx = \int_A^0 (-kx)\,dx$$

$$= -\frac{kx^2}{2}\Bigg|_A^0$$

$$= 0 - \left(-\frac{kA^2}{2}\right) = \frac{kA^2}{2}$$

In general for a horizontal spring:

$$W^{F_s}_{x_i x_f} = \int_{x_i}^{x_f} -kx\,dx = -\frac{kx^2}{2}\Bigg|_{x_i}^{x_f}$$

$$\boxed{W^{F_s}_{x_i x_f} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2}$$

Power $\equiv$ Rate at which a force does work

is defined as

$$\simeq \frac{\Delta W}{\Delta t}$$

If Work varies with time

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

$$[P] = J/s = W \text{ (watts)}$$

other unit: $1 \text{ hp} = 550 \frac{\text{ft lbs}}{\text{s}} = 746 \text{ W}$

"horsepower"

$$P = \frac{dW}{dt}$$

If force is constant

$$W = \vec{F} \cdot \int d\vec{r} = \vec{F} \cdot \vec{r}$$

$$\boxed{\frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}} = P$$

(see Prob. #5 for Power example)

Quiz:

$\vec{F} = (4x\,\vec{i} + 3y\,\vec{j})$ N

acts on an object of mass 1 kg starting from rest, moving along x-direction from the origin to x=5m

a.) Find the work done by the force for this motion

b.) (This is the only force doing work) What is the final velocity of the object?