No office hours Tuesday or Wednesday

Instead Monday morning 11-12.AM

Tuesday - Dr. Rapp

- Examples of conservation of Momentum and Demo
- & start Ch. 15

Last time we ended with examples of Circular Motion

Circular Motion $r = R = \text{const.}$

$\frac{dr}{dt} = \frac{d^2r}{dt^2} = 0$

makes polar coordinates useful

$\sum F_r = ma_r$ and $\sum F_\theta = ma_\theta$

$a_r = -\omega^2 R$ and $a_\theta = R\alpha$

$\sum F_r = \ldots\ldots = -m\omega^2 R$

$\sum F_\theta = \ldots\ldots = mR\alpha$

may have $\sum F_z = N - mg = ma_z = 0$ ($\perp$ to circle)

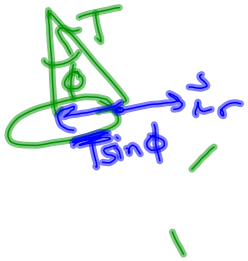
Circular motion is possible due to radial component(s) of force(s)

Examples: (from previous lectures)

$\sum F_r = -T = -m\omega^2 R$

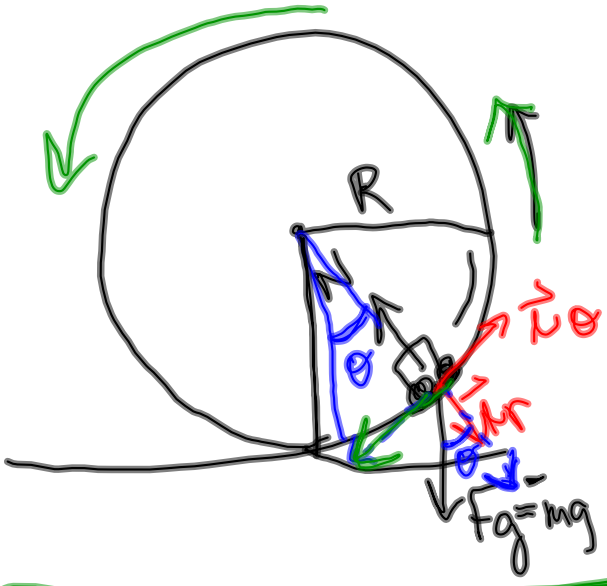
$\sum F_r = -T\sin\phi = -m\omega^2 R$

$\sum F_r = -f_{fric} = -m\omega^2 R$

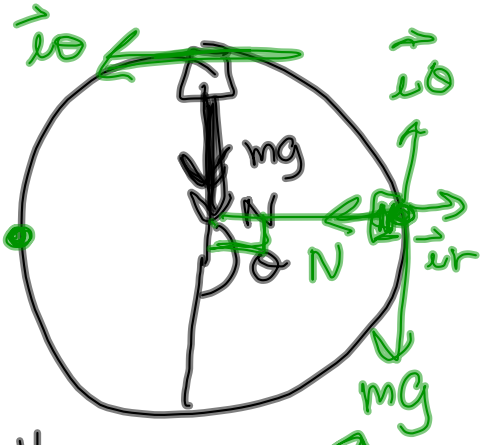


$\sum F_r = -\frac{Gm_E m}{r^2} = ma_r$

$= -m\omega^2 R$



on Earth

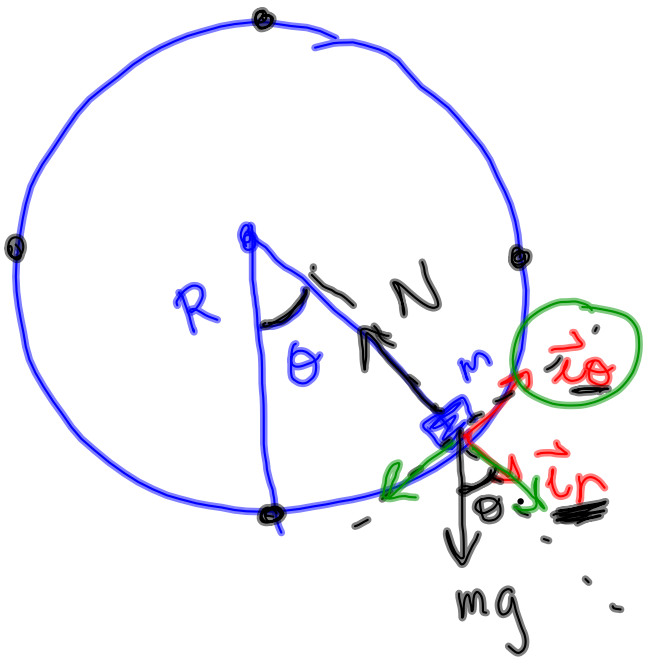


$\Sigma F_r = -N$

$\Sigma F_\theta = -mg$

$$\Sigma F_r = -N + mg\cos\theta = ma_r = -m\omega^2 R = -\frac{mV^2}{R}$$

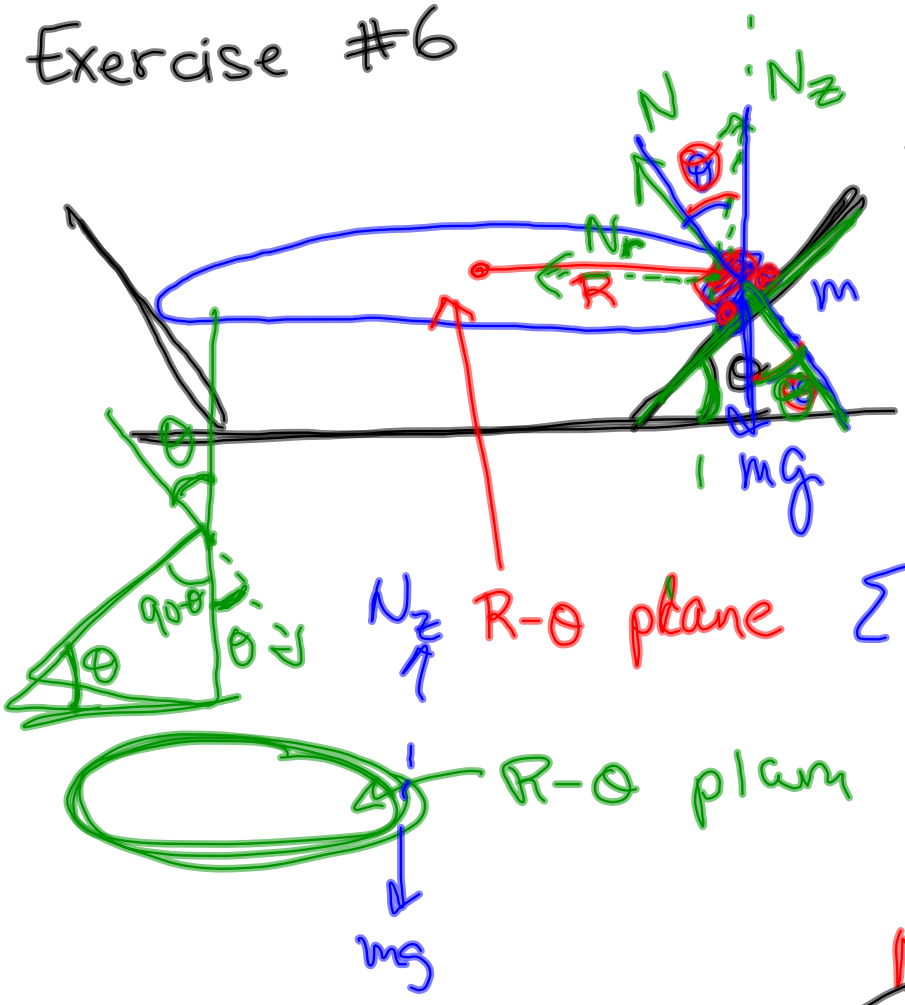
$$\Sigma F_\theta = -mg\sin\theta = ma_\theta = mR\alpha$$



$$\vec{F}_g = F_{gr}\vec{\imath}_r + F_{g\theta}\vec{\imath}_\theta$$

$$= mg\cos\theta\,\vec{\imath}_r - mg\sin\theta\,\vec{\imath}_\theta$$

Exercise #6



N_z R-θ plane $\sum F_z = N_z - mg = 0$

$N_z = mg$

$N\cos\theta = mg$

$N = \frac{mg}{\cos\theta}$

$\sum F_r = -N_r = -N\sin\theta = ma_r$

$= -m\omega^2 R$

$= -m\frac{v^2}{R}$

Given θ, what is ~~the~~ speed to make turn?

$-mg\tan\theta = -\frac{mv^2}{R}$

$v = \sqrt{gR\tan\theta}$

Ch. 14 Torque and Conservation of Angular Momentum

Motion

translational	rotational
kinematics: $\vec{r} = x\hat{i} + y\hat{j}$, $\vec{v}$, $\vec{a}$	$s = R\theta$, $\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d^2\theta}{dt^2}$
→ solve for $\vec{r}(t)$	→ can solve motion for $\theta(t)$
$\vec{F} = m\vec{a}$, $\vec{p} = m\vec{v}$	$\vec{\tau} = \vec{r} \times \vec{F} = I\alpha$ (cross product), $\vec{L} = \vec{r} \times \vec{p} = I\omega$
	$\vec{L}$: ang. momentum; I: (Ch. 15)

Goal: to be able to calculate cross product

Apply to defn's of $\vec{\tau}$ & $\vec{L}$

torque $\vec{\tau} = \vec{r} \times \vec{F}$

ang. momentum $\vec{L} = \vec{r} \times \vec{p}$

$= \vec{r} \times (m\vec{v})$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$ ← scalar

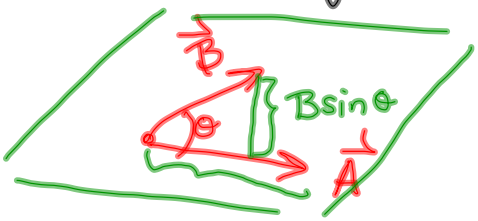
(Dot Product) $= |\vec{A}||\vec{B}|\cos\theta$

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta$ ← magnitude

(Cross Product)

direction is ⊥ to the plane that contains $\vec{A}$ & $\vec{B}$ determined by Right-hand-rule (RHR)

gives another vector $\vec{C}$



mag. = $AB\sin\theta$

direction is out of screen

$\vec{C}$ (by RHR) points out of screen

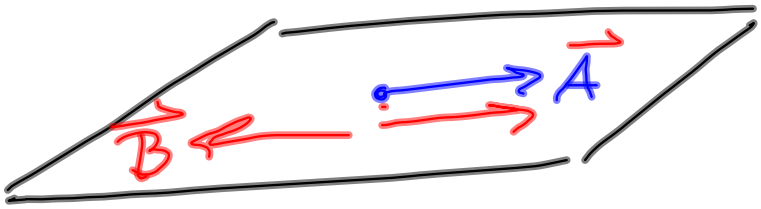
$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

$= -(\vec{B} \times \vec{A})$

RHR: place fingers (of right hand) along first vector w/ palm pointing from first to second vector ⇒ then thumb give direction of cross product

Fingers are curling counter clockwise or clockwise

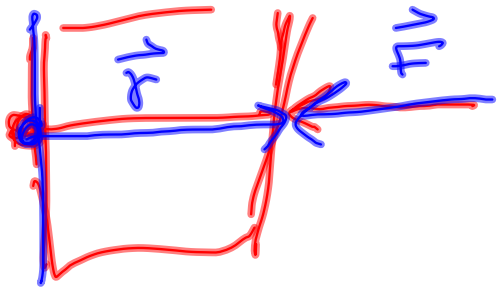
$\vec{\tau} = \vec{r} \times \vec{F}$



$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta = 0$$

if $\vec{A}$ & $\vec{B}$ are $\parallel$

then cross-product is 0



$\vec{r}$

Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{r}$: vector pt. from axis of rotation to where force is applied on object

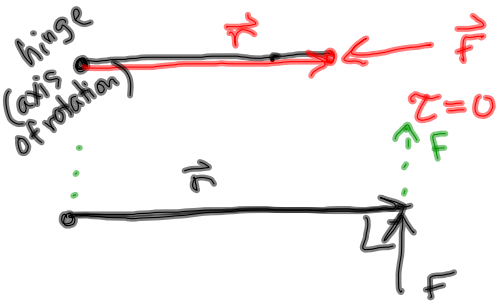
$\vec{\tau} = |\vec{r}||\vec{F}|\sin\theta$ (RHR)

= force times lever arm ($r\sin\theta$)

Examples. /

Top view of door (1.5 m wide)

$\vec{F} = 20\,N$ applied force



$\vec{\tau} = \vec{r} \times \vec{F}$

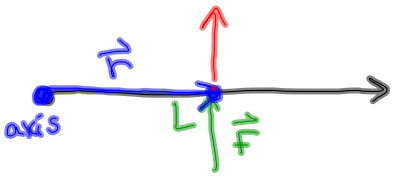
$= rF\sin\theta$

$= F(1.5m)\cdot 1$

$= 20N(1.5m)$

$= 30\,N\cdot m$

out of paper

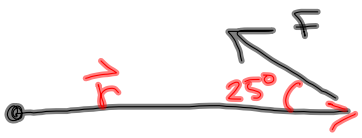


$\vec{\tau} = |r||F|\sin\theta$

$= (0.75m)(20N)\cdot 1 = 15\,N\,m$

out of paper

counter clockwise

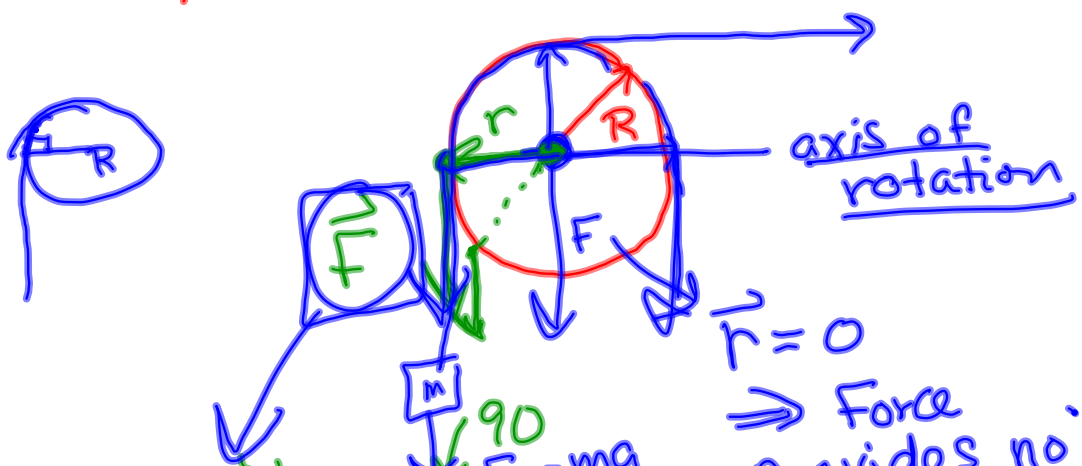


$\vec{\tau} = |\vec{r}||\vec{F}|\sin\theta = (1.5m)(20N)\sin 25$

$= 12.6\,N\cdot m$ out of paper

lever arm = $1.5\sin 25\,m$

Pulley:



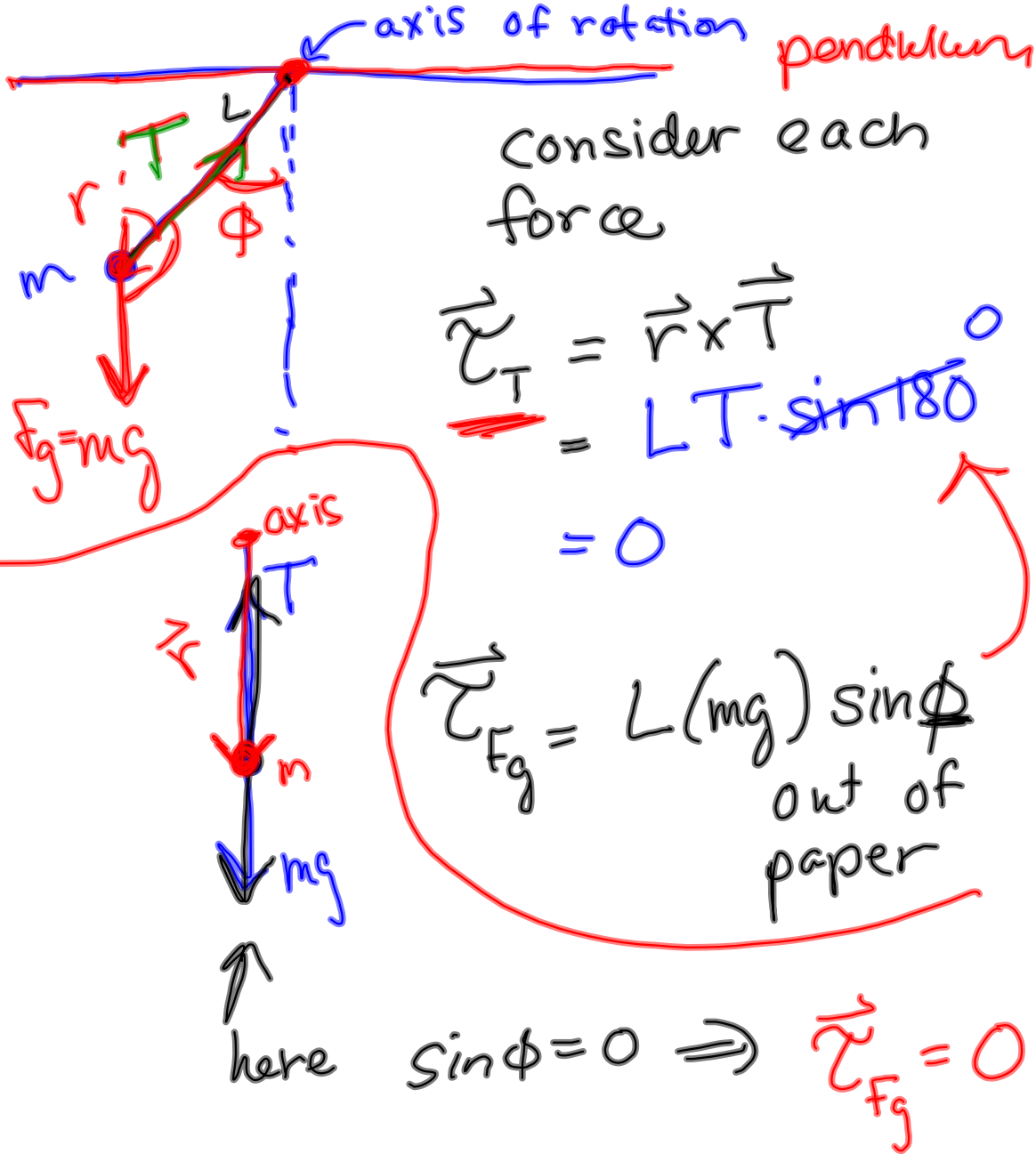
$\vec{r} = 0$

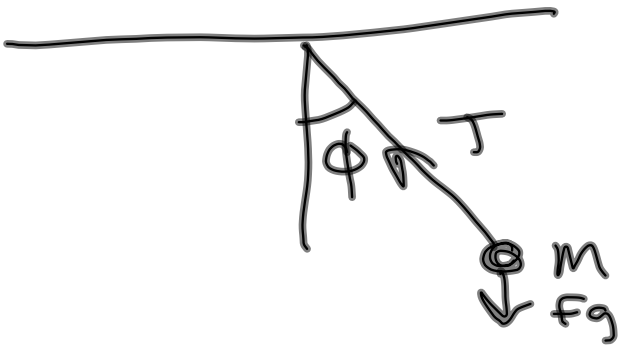
$\Rightarrow$ Force provides no torque (cannot cause rotation)

$\vec{\tau} = |\vec{r}||\vec{F}|\sin\theta$

$= \frac{RF \cdot 1}{}$

$= R \cdot mg$ out of paper (CC)





$\vec{\tau} = Lmg\sin\phi$ into paper

Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times (m\vec{v})$$

$$= m(\vec{r} \times \vec{v})$$

Angular momentum is calculated "with respect to some reference point" such as an axis of rotation.

Relationship between Angular Momentum and torque

Before: $\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$

$= \frac{d}{dt}(m\vec{v})$

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$

If $\sum \vec{F}_{ext} = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0$

$\vec{p}$ = constant

Conservation of momentum.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \vec{r} \times \left(\frac{d\vec{p}}{dt}\right)$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

$$= \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \vec{v} \times (m\vec{v}) + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

torque is $\frac{d}{dt}$ of ang. mom.

IF $\sum \vec{\tau}_{ext} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0$

$\vec{L}$ is constant

cons. of ang. momentum