Today: Review via Quiz from last time

- Circular Motion (Ch. 13)

---

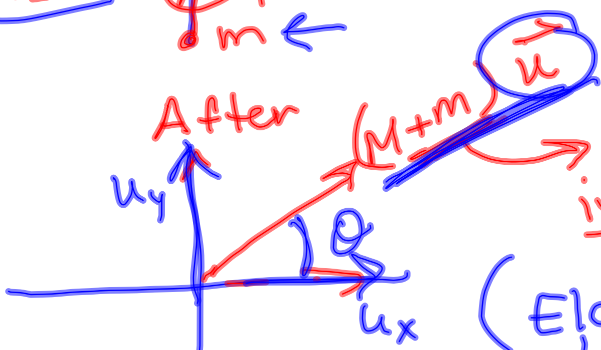
1. Before

If $\sum \vec{F}_{ext} = 0$

then $\vec{P}_i = \vec{P}_f$

$\vec{P}_i$: before coll.

$\vec{P}_f$: after collision



stick together

inelastic

(Elastic $\Rightarrow K_i = K_f$ → not this case)

$$\boxed{\vec{P}_i = \vec{P}_f}$$

$P_{ix} = P_{fx}$ and $P_{iy} = P_{fy}$

$$Mv_2 + \cancel{m(0)} = (M+m)u_x \quad (1)$$

$u_x = u\cos\theta$

$$M(0) + m(v_1) = (M+m)u_y \quad (2)$$

$u_y = u\sin\theta$

$$u_x = \frac{Mv_2}{M+m} \quad \text{and} \quad u_y = \frac{mv_1}{M+m}$$

$$u = \sqrt{u_x^2 + u_y^2} \quad \text{at} \quad \theta = \tan^{-1}\left(\frac{u_y}{u_x}\right)$$

relative to horizontal

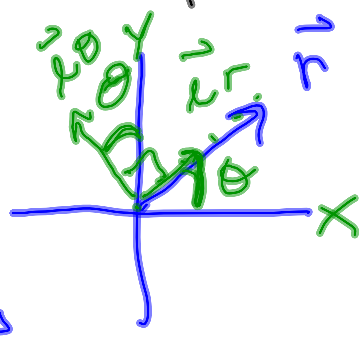
2. Start by drawing $\vec{\imath}_r$ & $\vec{\imath}_\theta$, write down $\vec{\imath}_r$ & $\vec{\imath}_\theta$ in terms of $\vec{\imath}$ & $\vec{\jmath}$.

calculate $\frac{d\vec{\imath}_r}{dt}$ and $\frac{d\vec{\imath}_\theta}{dt}$.

Final Goal: Starting with

$\vec{r} = x\vec{\imath} + y\vec{\jmath}$ old coordinate system

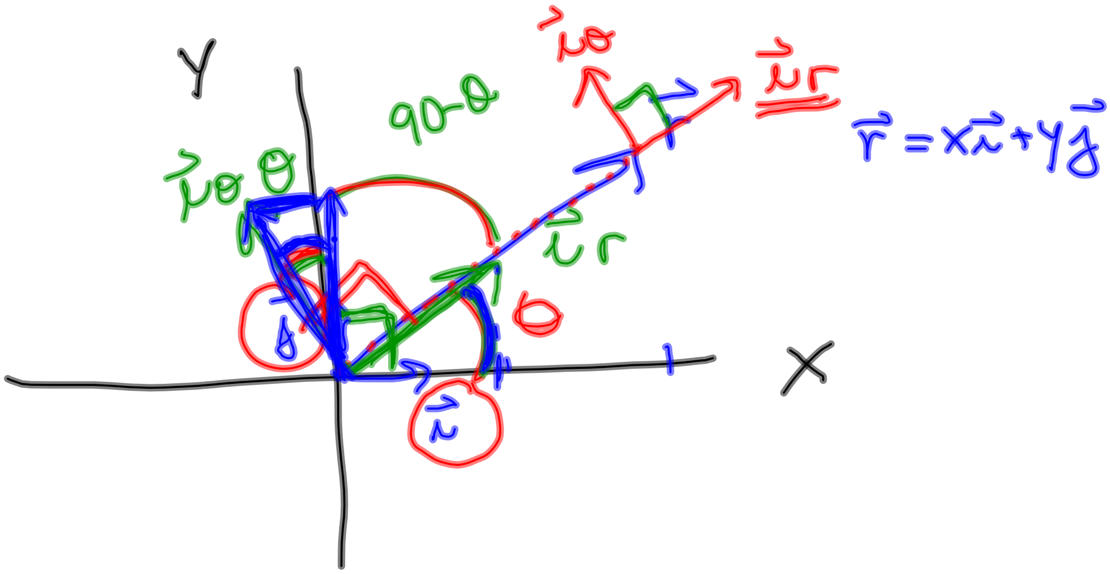
$\vec{r} = r\vec{\imath}_r$



Write

$\vec{V} = V_r\vec{\imath}_r + V_\theta\vec{\imath}_\theta$

and $\vec{a} = a_r\vec{\imath}_r + a_\theta\vec{\imath}_\theta$



$$\vec{u}_r = 1 \cdot \cos\theta\, \vec{i} + \sin\theta\, \vec{j}$$

$$\vec{u}_\theta = -\sin\theta\, \vec{i} + \cos\theta\, \vec{j}$$

$$\frac{d\vec{u}_r}{dt} = \frac{d\vec{u}_r}{d\theta} \cdot \frac{d\theta}{dt} \cdot \text{ Chain Rule}$$

$$= -\sin\theta \frac{d\theta}{dt} \vec{i} + \cos\theta \frac{d\theta}{dt} \vec{j}$$

$$\boxed{\frac{d\vec{u}_r}{dt} = \frac{d\theta}{dt}\left(-\sin\theta\, \vec{i} + \cos\theta\, \vec{j}\right) = \frac{d\theta}{dt} \vec{u}_\theta}$$

$$\frac{d\vec{u}_\theta}{dt} = \frac{d\vec{u}_\theta}{d\theta} \frac{d\theta}{dt}$$

$$= -\cos\theta \frac{d\theta}{dt} \vec{i} - \sin\theta \frac{d\theta}{dt} \vec{j}$$

$$\boxed{\frac{d\vec{u}_\theta}{dt} = \frac{d\theta}{dt}\left(-\cos\theta\, \vec{i} - \sin\theta\, \vec{j}\right) = -\frac{d\theta}{dt} \vec{u}_r}$$

After showing $\frac{d\vec{i}_r}{dt} = \frac{d\theta}{dt}\vec{i}_\theta$

and $\frac{d\vec{i}_\theta}{dt} = -\frac{d\theta}{dt}\vec{i}_r$

can plug into equations in derivation:

$\vec{r} = r\vec{i}_r$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\vec{i}_r + r\frac{d\vec{i}_r}{dt}$$ (Product Rule)

Goal $\vec{v} = v_r\vec{i}_r + v_\theta\vec{i}_\theta$

$$\vec{v} = \underbrace{\boxed{\frac{dr}{dt}\vec{i}_r}}_{v_r} + \underbrace{\boxed{r\frac{d\theta}{dt}\vec{i}_\theta}}_{v_\theta}$$

$\vec{a} = \frac{d\vec{v}}{dt}$ = 2 terms + 3 terms

$$= \frac{d^2r}{dt^2}\vec{i}_r + \frac{dr}{dt}\boxed{\frac{d\vec{i}_r}{dt}} + \dots$$

$\vec{r} = r\vec{i}_r$

$\frac{d\vec{r}}{dt} = \frac{dr}{dt}\vec{i}_r + r\frac{d\vec{i}_r}{dt}$

length

direction

Plug in $\frac{d\theta}{dt}\vec{i}_\theta$

$$+ \frac{dr}{dt}\frac{d\theta}{dt}\vec{i}_\theta + r\frac{d^2\theta}{dt^2}\vec{i}_\theta + r\frac{d\theta}{dt}\boxed{\frac{d\vec{i}_\theta}{dt}}$$

Plug in $-\frac{d\theta}{dt}\vec{i}_r$

collect $\vec{i}_r$ terms & $\vec{i}_\theta$ terms

$\Rightarrow \vec{a} = a_r\vec{i}_r + a_\theta\vec{i}_\theta$

Result :

$$\omega = \frac{d\theta}{dt}$$

ang. vel.

$$\vec{v} = \frac{dr}{dt}\,\vec{i}_r + r\omega\,\vec{i}_\theta$$

and
$$\vec{a} = \left[\frac{d^2r}{dt^2} - r\omega^2\right]\vec{i}_r + \left[2\frac{dr}{dt}\omega + r\alpha\right]\vec{i}_\theta$$

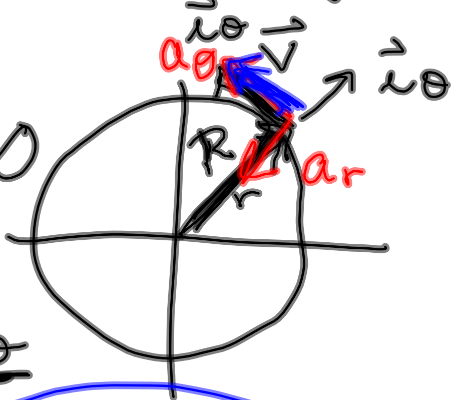
$$\alpha = \frac{d^2\theta}{dt^2}$$

ang. acc.

Circular Motion

$r = R = \text{const.}$

$\frac{dr}{dt} = 0$ and $\frac{d^2r}{dt^2} = 0$



$$\vec{v} = R\omega\,\vec{i}_\theta$$

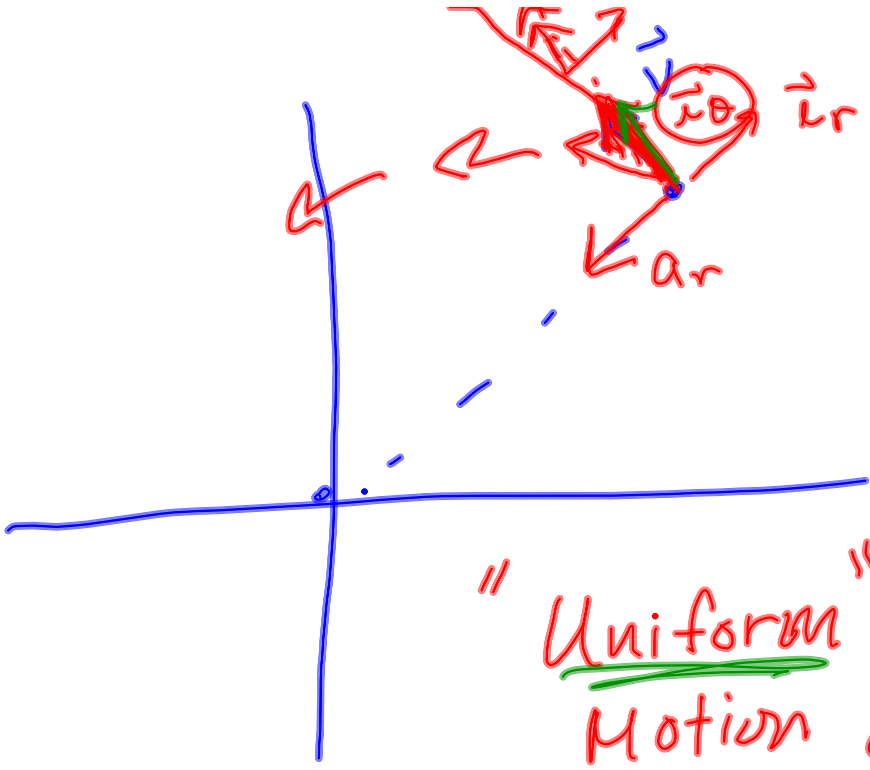
$$\vec{a} = -R\omega^2\,\vec{i}_r + R\alpha\,\vec{i}_\theta$$

changes direction of $\vec{v}$

"centripetal acc" = radial acc.

changes magnitude of $\vec{v}$

"tangential" acc.



"Uniform" Circular Motion $\frac{d\omega}{dt} = 0$

$\omega = \text{const}$ $\quad \alpha = 0$

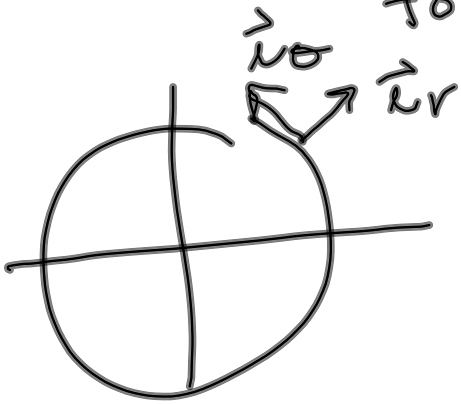
$\sum F_r = -m\omega^2 R$ $\qquad \sum F_\theta = 0$

$$\sum \vec{F} = m\vec{a} = m a_r \vec{i}_r + m a_\theta \vec{i}_\theta$$

$$\sum F_r = m a_r \quad \text{and} \quad \sum F_\theta = m a_\theta$$

$$\boxed{\sum F_r = -m\omega^2 R} \quad \text{and} \quad \sum F_\theta = mR\alpha$$

for $r = R = \text{const}$

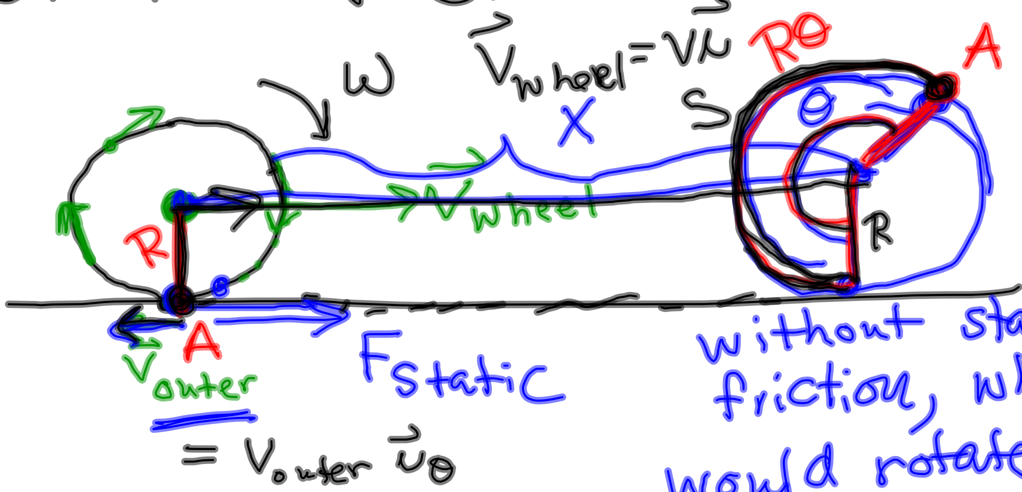


What causes circular motion?

- Force(s) with a net (non-zero) radial component $= -m\omega^2 R$

Tension, friction, gravity, normal force

Static friction



without static friction, wheel would rotate in place.

Static friction allows rolling w/out slipping

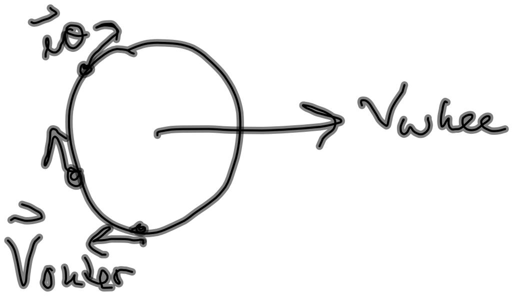
$2\pi R$ = full turn (circumference

$R\theta = S$ = arc length through which wheel turned

If rolled without slipping, $x = \underline{R\theta}$

$$\vec{V}_{wheel} = \frac{dx}{dt}\,\vec{i} = \frac{d}{dt}(R\theta)\,\vec{i} = R\frac{d\theta}{dt}\,\vec{i} = R\omega\,\vec{i}$$

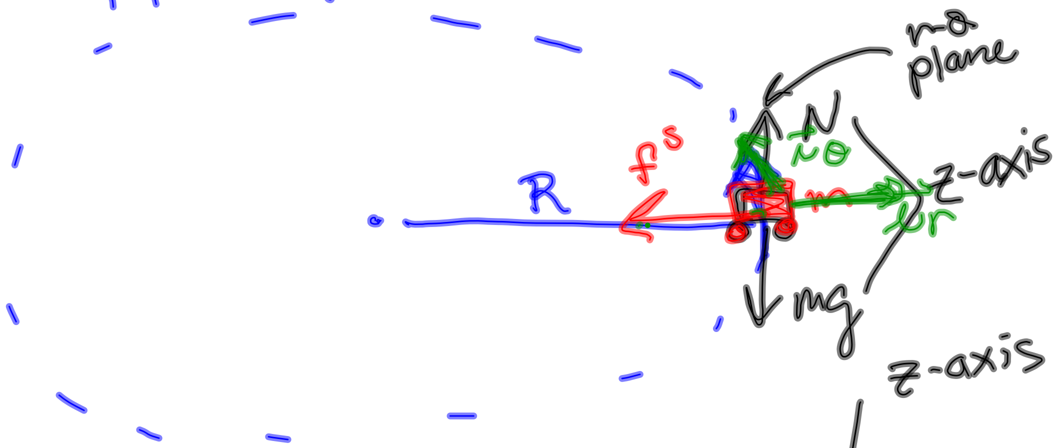
$$\vec{V}_{outer} = \frac{dr}{dt}\,\vec{e}_r + R\omega\,\vec{e}_\theta$$

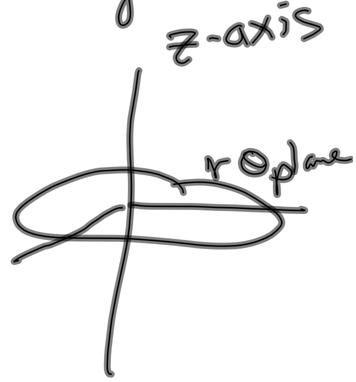


$$|\vec{V}_{outer}| = |\vec{V}_{wheel}|$$

Static friction (no sliding)

If wheels turned, can have component of friction in radial direction





curve of radius R
coeff. of friction is μ

$$\sum \underline{F_r} = -m\omega^2 R = m a_{\underline{r}}$$

$$-f = -m\omega^2 R \qquad f_s^{max} = \mu N$$

What is max. speed w/ which you can make curve?

$$f \le \mu N \Rightarrow -m\omega^2 R \le \mu m g$$

$$\omega \le \sqrt{\frac{\mu g}{R}} \qquad v = \omega R$$

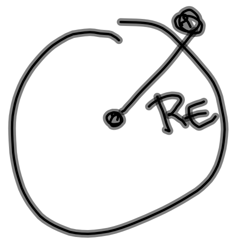
$$v \le \sqrt{\mu g R}$$

Gravity

due to Earth

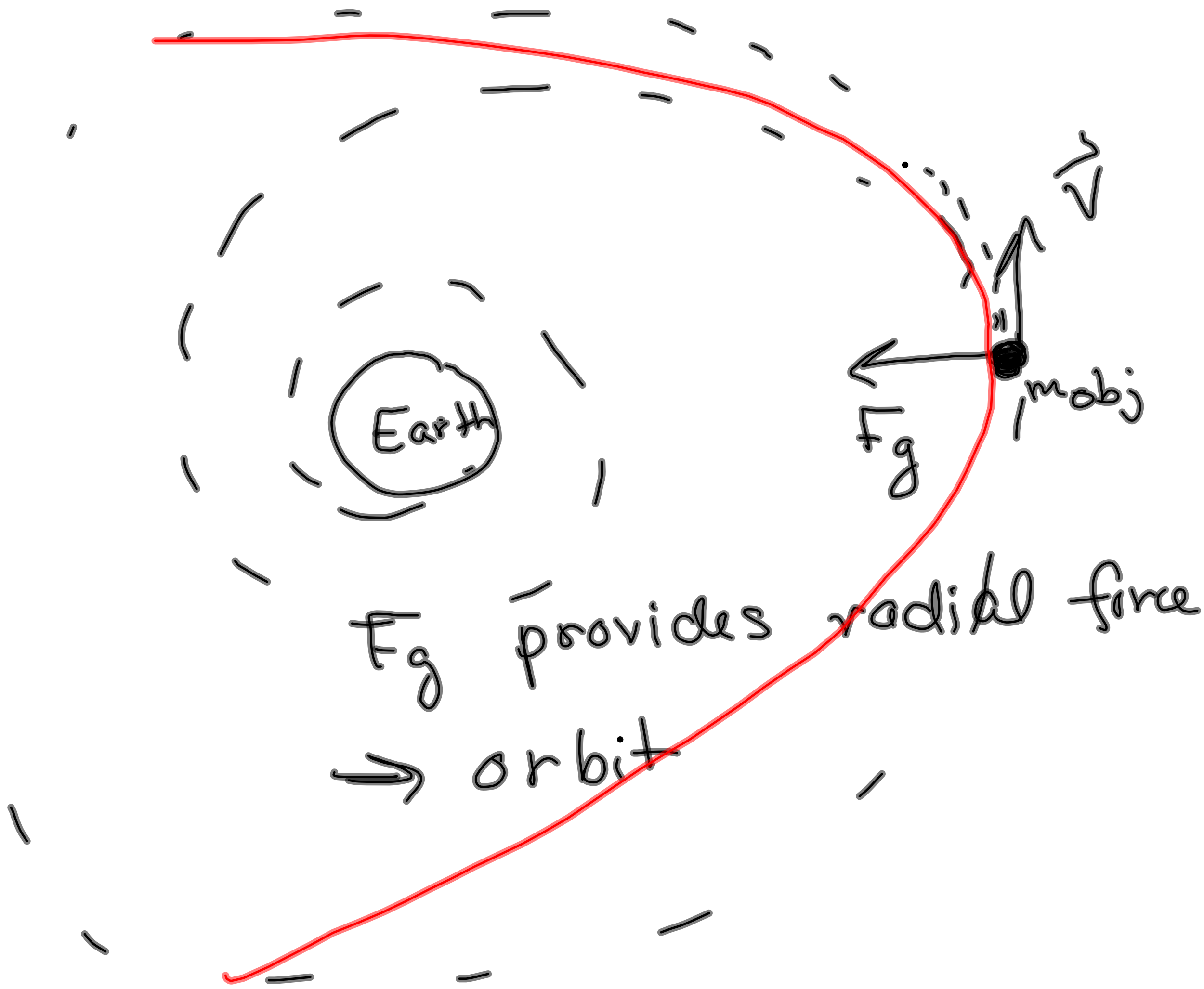
$$\boxed{F_g = -\frac{G m_E \, m_{obj}}{R^2}}$$

If ~~you~~ object is on surface of Earth



$$\Rightarrow F_g = -\underbrace{\frac{G m_E}{R_E^2}}_{9.8 \frac{m}{s^2}} m_{obj}$$

$$F_g = -m_{obj}\, g$$

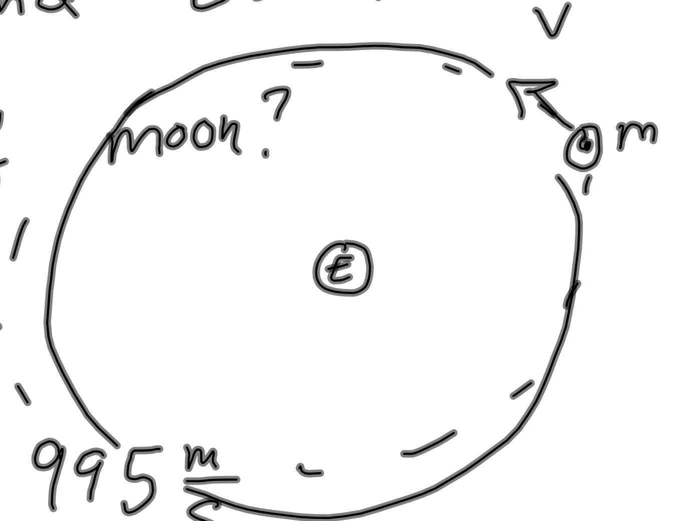

$\vec{v}$
m_{obj}
F_g
Earth
F_g provides radial force
→ orbit

Problem #2 (Example)

$R = 3.8\times10^{5}\ \text{km} \times \frac{10^{3}\ \text{m}}{1\ \text{km}} = 3.8\times10^{8}\ \text{m}$

period of revolution $= T = 2.4\times10^{6}\ \text{s}$

moon around Earth.



a.) velocity of moon?

$v = \frac{2\pi R}{2.4\times10^{6}\ \text{s}} = 995\ \frac{\text{m}}{\text{s}}$ $2\pi R = 1$ rev.

b.) $\sum F_r = -\frac{G m_E m_M}{R^2} = m a_r$

$= -m\omega^2 R = -\frac{m v^2}{R}$

$\vec{a} = a_r \vec{e}_r = -\frac{v^2}{R} = \frac{-(995)^2}{(3.8\times10^{8}\ \text{m})} =$

$= -2.6\times10^{-3}\ \frac{\text{m}}{\text{s}^2}\ \vec{e}_r$

c.) solve for m_E

$m_E = \frac{a R^2}{G} = 5.7\times10^{24}\ \text{kg}$