Next week office hours:

T  11-12
W  10-11
R  10-11 (unchanged)
Review Ch. 8

Force is conservative if
1) Work done by force in a closed path is 0.
2) \( \exists \) a function \( U(x,y) = \text{"potential energy"} \) such that the force can always be described as:
\[
F = -\frac{\partial U}{\partial x} \hat{i} - \frac{\partial U}{\partial y} \hat{j}
\]
\[
F_x = -\frac{\partial U}{\partial x} \quad \text{and} \quad F_y = -\frac{\partial U}{\partial y}
\]

If 1-dim
\[
F_x = -\frac{\partial U}{\partial x} \Rightarrow U(x) = \int F_x \, dx + C
\]
Examples of conservative forces in nature: Spring force + gravity

1) \( \vec{F}^s = -kx \ \hat{i} \) (horizontal spring)

In general, \( X_{eq} \neq \) constant

\[ \vec{F}^s = -K(X - X_{eq}) \ \hat{i} \]

\( L \) = displacement from equilibrium position.

\[ U(x) = -\int F^s \, dx = \frac{1}{2} kx^2 - kx_{eq} x + C \]

\[ F = -kx \]

\[ U = \frac{1}{2} kx^2 = \frac{1}{2} \left( kX^2 - (2kx_{eq}X + 2C) \right) = \frac{1}{2} k(X - A)^2 \]

2) \( \vec{F} = -mg \ \hat{j} \)

\[ F_y = -\frac{dU}{dy} \]

\[ U(y) = -\int F_y \, dy = -\int -mg \, dy = mg y + C \]
$W_{\text{total}} = K_f - K_i$  always true

If only conservative force doing work, then total mechanical energy $(K+U)$ is conserved.

$K_i + U_i = K_f + U_f$

$W_{\text{cons}} = \Delta U = -(U_f - U_i) = U_i - U_f$

$U_i - U_f = K_f - K_i$

$U_i + K_i = U_f + K_f$
Last year's Exam:

Problem 2:

\[ \mathbf{F}_{sp} = (-kx - \alpha x^3) \hat{r} \]

a.) \( U = -\int F_{sp} \, dx \)

\[ W_{x_i, x_f}^{F_{sp}} = \int_{x_i}^{x_f} (-kx - \alpha x^3) \, dx \]

\[ = -\frac{kx^2}{2} - \frac{\alpha x^4}{4} \bigg|_{x_i}^{x_f} = -\frac{kx_f^2}{2} - \frac{\alpha x_f^4}{4} + \frac{kx_i^2}{2} + \frac{\alpha x_i^4}{4} \]

\[ = -(U(x_i) - U(x_f)) \]

\[ \Rightarrow U(x) = \frac{kx^2}{2} + \frac{\alpha x^4}{4} + C \]

\[ F_x = -\frac{dU}{dx} = -kx - \alpha x^3 \]
b.) Find max. compression $a$.
- can use work-energy theorem
  \[ W_{\text{total}} = K_f - K_i \]
  \( b/c \) you know \( \Delta K : K_i = \frac{1}{2} m v_0^2 \)
  \( K_f = 0 \), and \( W_{\text{total}} \) is a fun.
  \( \Rightarrow \) at max compression \( v_f = 0 \)!

\[ \begin{align*}
   W_{\text{total}} &= K_f - K_i = 0 \\
   W_{\text{total}} &= W_{F_5} = -\frac{Kx^2}{2} - \frac{a^4}{4} \\
   -\frac{ka^2}{2} - \frac{a^4}{4} &= -\frac{1}{2} m v_0^2 \\
   \text{solve for } a.
\end{align*} \]
OR use conservation of mech. energy:

\[ U_i + K_i = U_f + K_f \]

\[ K = \frac{1}{2} mv^2 \quad \text{and} \quad U(x) = \frac{kx^2}{2} + \frac{ax^4}{4} \]

\[ 0 + \frac{1}{2} mv_0^2 = \frac{ka^2}{2} + \frac{a^4}{4} + 0 \]

\[ \text{solve for } a \]

\[ U_i = U(x=0) = 0 \]

\[ U_f = U(x=a) = \frac{ka^2}{2} + \frac{a^4}{4} \]

\[ K_i = \frac{1}{2} mv_0^2 \]

\[ K_f = \frac{1}{2} m(0)^2 = 0 \]
Past Exam #3:

starts from rest

spring is compressed a distance $A$.

Force of a spring when $x_{eq} \neq 0$

$F^{sp} = -k(x-x_{eq}) = -k(x-A)$

a.) no friction. Determine maximum height $h$ that it will reach.

At max. height $v=0$

Use $W_{\text{total}} = K_f - K_i$ b/c change in kinetic energy is known $\frac{1}{2}$

Work done will be a fcn. of height $h$.

$W_{\text{total}} = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0$
\[ W_{\text{total}} = W_{\text{fg}} + W_{\text{fs}} + W_{\text{h}} = 0 \]

\[ W_{\text{fg}} = \int_{0}^{x_f} -mg \sin \theta \, dx + \int_{0}^{h} -mg \cos \theta \, dx \]

\[ \sin \theta = \frac{h}{x_f} \]

\[ \Rightarrow x_f = \frac{h}{\sin \theta} \]

\[ = -mg \sin \theta \times \int_{0}^{x_f} \frac{h}{\sin \theta} = -mg \frac{h^2}{\sin \theta} \]

Spring will only affect motion until it reach equil. then it stops (not attached to mass)

\[ W_{\text{fs}} = \int_{0}^{A} -k(x-A) \, dx = \int_{0}^{A} (-kx + kA) \, dx \]

\[ = -\frac{kx^2}{2} + kAx \bigg|_0^A = -\frac{kA^2}{2} + kA^2 \]

\[ = \frac{kA^2}{2} \]
\[ W_{total} = -mg h + \frac{1}{2} kA^2 = 0 \]
\[ h = \frac{KA^2}{2mg} \]

b.) If it reaches height \( h' \) instead, and \textbf{there is} friction, determine the coefficient of friction \( \mu \).

\[ F_f = \mu N \]
\[ \Sigma F_y = N - F_{gy} = 0 \]
\[ N = mg \cos \alpha = F_f = \mu mg \cos \alpha \]
\[ W = \int_0^{h'/\sin \alpha} (-\mu mg \cos \alpha) dx \]
\[ = -\mu mg \cos \alpha \cdot \frac{h'}{\sin \alpha} \]
\[ = -\mu mg h' \cos \alpha \frac{1}{\sin \alpha} = -\mu mg h' \cot \theta \]

\[ W_{total} = W_f + W_g + W_s + W_n = 0 \]
\[ = -\mu mg h' \cot \theta - mg h' + \frac{1}{2} kA^2 = 0 \]

Solve for \( \mu \) 
(\( h' \) given)
Last quiz:

\[ F(x) = \frac{\alpha m}{x^2} = \alpha m x^{-2} \]

\[ W\left|_a^b \right. = \int_a^b F(x) \, dx = \int_a^b \alpha m x^{-2} \, dx \]

\[ = -\alpha m \left( \frac{1}{b} - \frac{1}{a} \right) \]

b.) given \( v(x=a) = v_0 \)

Find \( v(x=b) \)

\[ W_{total} = K_f - K_i = \frac{1}{2} mv_b^2 - \frac{1}{2} mv_0^2 \]

Why can't you solve

\[ F = ma \quad \Rightarrow \quad \int a \, dt \]

\[ = \int \frac{\alpha m}{x^2} \, dt \]

\[ = \int \left[ x(t) \right]^2 \, dt \]

If you don't have \( a(t) \), then don't know how to integrate w.r.t. \( t \)!