Ch. 7

Work:

\[ W_{F} = \int_{c_{1}}^{c_{2}} \mathbf{F} \cdot d\mathbf{r} = \int_{x_{1}}^{x_{2}} F_{x} \, dx + \int_{y_{1}}^{y_{2}} F_{y} \, dy \]

\[ \mathbf{F} = (4x^{2} + 3y) \delta \mathbf{N} \]

\[ W = \int_{0}^{5} 4x \, dx + \int_{0}^{2} 3y \, dy \]

\[ W_{\text{total}} = K_{f} - K_{i} \]

\[ K = \frac{1}{2} mv^{2} \]
Kinetic energy is energy of a moving object.

Ch. 8
Energy that an object has with the potential to move
Potential Energy is associated with a "conservative force"

- What is a conservative force?
- How to calculate the Potential energy function $U(x)$ or $U(y)$

- $K + U = \text{total mechanical energy}$

When is there conservation of total mechanical energy?
- A force is **conservative** if the work done by the force on an object moving from \( \vec{r}_1 \) to \( \vec{r}_2 \) is independent of the path taken.

  work done around a closed path is 0.

\[
W_{\vec{r}_1, \vec{r}_2} = 0
\]
Example of conservative vs. non-conservative forces:

Problem #1 (Ch. 8)

Path from \( x_1 \rightarrow x_2 \rightarrow x_1 \)
mass \( m \), coefficient of friction \( \mu \)

\[
f_{\text{fric}} = \mu N = \mu mg
\]

\[
W_{x_1, x_2} = \int_{x_1}^{x_2} f_{\text{fric}} \, dx = \int_{x_1}^{x_2} -\mu mg \, dx
\]

\[
= -\mu mg (x_2 - x_1)
\]

\[
W_{x_2, x_1} = \int_{x_2}^{x_1} f_{\text{fric}} \, dx = \mu mg \int_{x_2}^{x_1} \, dx
\]

\[
= \mu mg (x_1 - x_2)
\]

\[
= -\mu mg (x_2 - x_1)
\]

\[
W_{x_1, x_1} = -2\mu mg (x_2 - x_1) \neq 0
\]
See problem # 4 (Ch. 7) (upward)

\[ W_{Fg}^{\text{upward}} = -mgh \]

\[ W_{Fg}^{\text{downward}} = +mgh \]

\[ W_{\text{closed path}}^{Fg} = 0 \]
The Potential Energy Function

to determine whether a force is conservative

Consider only 1-d motion:

\[ W_{x_1, x_2} = \int_{x_1}^{x_2} F_x \, dx \]

If \( F_x = \frac{d}{dx} h(x) \), then

\[ W = \int_{x_1}^{x_2} \frac{d}{dx} h(x) \, dx = h(x) \bigg|_{x_1}^{x_2} = h(x_2) - h(x_1) \]

\( U(x) \) is potential energy fcn.

\[ U(x) = -h(x) \]

\( F_x = -\frac{d}{dx} U(x) \) (only works for a conservative force)

\[ F_x = \int_{x_1}^{x_2} \frac{d}{dx} U(x) \, dx = -U(x) \bigg|_{x_1}^{x_2} = -(U(x_2) - U(x_1)) = U(x_1) - U(x_2) \]

Recall:

\[ W_{fs} = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2 \]
Work done by F is
\[-\Delta U = U(x_1) - U(x_2)\]

If \( U(x, y) \), then
\[ F_x = -\frac{\partial U}{\partial x} \quad \text{and} \quad F_y = -\frac{\partial U}{\partial y} \]
\[ \vec{F} = -\frac{\partial U}{\partial x} \hat{i} - \frac{\partial U}{\partial y} \hat{j} \]

\[ \text{Partial derivative} \] (only vary one variable; leave others constant)

\[ F = 4x^2y^3 \]

\[ \frac{\partial F}{\partial x} = 8xy^3 \]
\[ \frac{\partial F}{\partial y} = 12x^2y^2 \]
Example:
\[ \vec{F}_g = m \vec{g} - \vec{0} \]
1. calculate \( W_{x_1,x_2} = \int_{x_1}^{x_2} F_x \, dx \)
2. set that equal to \( U(x_1) - U(x_2) = -\left[ U(x_2) - U(x_1) \right] \)
3. Find \( U(x) \rightarrow \text{potential energy function} \)
4. Check whether \( F_x = -\frac{dU}{dx} \)
5. Is \( -\frac{dU}{dx} \) the (x-component) force under all circumstances?

Is any other information needed to specify the force?
If no other info needed, then \( F \) is conservative
① \( W_{Fg}^{y_2} = \int_{y_1}^{y_2} F_g \, dy \)

\[
= \int_{y_1}^{y_2} -mg \, dy = -mg(y_2 - y_1)
\]

② \[ = -[U(y_2) - U(y_1)] \]

③ \( U(y) = mg'y \)

④ \[ F_y = -\frac{dU}{dy} = -mg \]

⑤ always the force
\[ F_{\text{fric}} = \mu N \rightarrow \text{not conservative} \]

need additional information of direction of motion.

Spring force: \( \vec{F}_s = -kx \vec{i} \)

\[ W_{x_1x_2}^{F_s} = \int_{x_1}^{x_2} (-kx)dx \]
\[ = - \left[ \frac{kx^2}{2} \right]_{x_1}^{x_2} \]
\[ = - \left[ \frac{kx_2^2}{2} - \frac{kx_1^2}{2} \right] \]
\[ = - \left[ U(x_2) - U(x_1) \right] \]

3. \( U(x) = \frac{kx^2}{2} \)

4. Check:
\[ F_x = -\frac{dU}{dx} = -kx \checkmark \]

5. This is always the spring force.

Whether spring is moving to the right or to the left
\[ F_x = -kx \]

\[ \therefore F_s \text{ is a conservative force and } U(x) = \frac{1}{2}kx^2 \]

is the potential energy associated w/ spring force.
Now know kinetic + potential energies.

Remember:

\[
W_{r_i \rightarrow r_f}^{\text{tot}} = K_f - K_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2
\]

\( \Rightarrow \) always true

If only conservative forces are doing work(!?),

\[
W_{r_i \rightarrow r_f}^{\text{tot}} = W_{r_i \rightarrow r_f}^{\text{cons}} = - [U_f - U_i]
\]

\[
W_{r_i \rightarrow r_f}^{\text{tot}} = U_i - U_f
\]

\[\Rightarrow U_i - U_f = K_f - K_i\]

\[\Rightarrow U_i + K_i = U_f + K_f\]

Conservation of total mechanical energy!
Quiz: Exercise #7
(Ch. 7)

\[ F(x) = \frac{\alpha m}{x^2} \quad (1\text{-dim}) \]

+ x

a.) Calculate work done by \( F \) as object moves from \( x=a \) to \( x=b \).

b.) \( F \) is only force. \( v(x=a) = v_o \). Calculate \( v(x=b) \).