Exam Tuesday RICHT 114
7-8:10 pm Sections 814 + 815 (I-Z)
8:15-9:25 Sections 813 + 815 (A-H)

Office Hours
F: 2-4 pm
M: 2-3, 4-6 pm
Review for Exam

1) \( \Sigma \vec{F} = m\vec{a} \): \( \Sigma F_x = ma_x \) \& \( \Sigma F_y = ma_y \)

- Isolate each object & draw the free-body for each
- \( \Sigma F_x = ma_x \) \& \( \Sigma F_y = ma_y \) for each object
  (considering that acc. of one object may be equal to that of other object)
-solve for the requested unknown in terms of the knowns

# 1 from 2007 Exam

Given: \( m_1, m_2 \) 
coefficient of friction \( \mu \)

\( m_1 \) moves down incline

b) Find acceleration of \( m_1 \)
a) Draw free-body diagram for each mass
b.) $\Sigma F_{y_1} = N - m_1 g \cos \theta = ma_y = 0$

$N = m_1 g \cos \theta$

$F_f = \mu m_1 g \cos \theta$

$\Sigma F_{x_1} = m_1 g \sin \theta - T - F_f = m_1 a$
\[ \Sigma F_{y_2} = T - m_2 g = m_2 a_y \]

\[ T = m_2 g + m_2 a \]

Plug \( T \) back into eqn. \( \Sigma F_{x_1} \):

\[ m_1 g \sin \theta - \mu m_1 g \cos \theta - (m_2 g + m_2 a) = m_1 a \]

\[ m_1 g \sin \theta - \mu m_1 g \cos \theta - m_2 g = (m_1 + m_2) a \]

\[ a_x = \frac{m_1 g \sin \theta - \mu m_1 g \cos \theta - m_2 g}{m_1 + m_2} \]

\( a_y \) for obj 2 = \( a_x \) for obj 1 \( \equiv a \)
2) Work-Energy Theorem

\[ W^{\text{tot}} = \Delta K.E. = K_f - K_i \]

- Start with free-body diagram
- For each force, calculate the work as object moves from some initial \( x_i \) to final \( x_f \)
- Add up total work of all forces
- Set equal to \( \Delta K.E. \)

\[ W^{\text{tot}} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \]
#2 (old exam)

Where does it turn around?

\[ v_f = 0 \]

\[ F = -\frac{\alpha}{x^2} \]

coeff. of friction \( \mu \)

\[ W^{\text{tot}} = K_f - K_i \]
\[ = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \]
\[ = 0 - \frac{1}{2} m v_i^2 \]
\[ W^{\text{tot}} = W^F + W^{f_f} + W^0 + W^g_0 \]

\[ W^F = \int_A^{xf} \frac{d}{x^2} \, dx \]

\[ = \frac{\alpha}{x} \bigg|_A^{xf} = \frac{\alpha}{xf} - \frac{\alpha}{A} \]

\[ W^{f_f} = \int_A^{xf} F_f \, dx \Rightarrow F_f = \frac{N}{mg} \]

\[ = \int_A^{xf} -\mu mg \, dx = -\mu mg \bigg|_A^{xf} \]

\[ = -\mu mg (xf - A) \]

\[ W^{\text{tot}} = K_f - K_i = -\frac{1}{2} mv_1^2 \]

\[ \frac{\alpha}{xf} - \frac{\alpha}{A} - \mu mg (xf - A) = -\frac{1}{2} mv_1^2 \]

Solve for \( x_f \)
b.) velocity when back at \( x = A \)?

\[
W_{x=A,x_f,A}^F = 0 \quad \text{b/c } F \text{ is conservative}
\]

\[
W_{x=A,x_f,A}^{frc} = -2\mu mg (x_f - A)
\]

\[
W_{x=A\rightarrow x_f\rightarrow A}^{tot} = -2\mu mg (x_f - A) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2
\]

Solve for \( v_f \)
\[ W_{\text{tot}} = K_f - K_i = \frac{1}{2}mv_f^2 - 0 \]

\[ a = \frac{v_f^2}{2x} \]

\[ f_1 = C_1 + C_2 \]

Initially compressed by distance \( x \).

\( \text{Solving for } t \)
\[ W_N = W_g = 0 \quad \text{and} \quad F_s = -kx \]

\[ W_{Fs} = \int_{-A}^{0} -kx \, dx \]

\[ = -\frac{1}{2} kx^2 \bigg|_{-A}^{0} = 0 - (-\frac{1}{2} kA^2) \]

\[ W_{Fi} = \int_{-A}^{0} c_1 \, dx + \int_{0}^{A} c_2 \, dy = c_1 x \bigg|_{-A}^{0} \]

\[ = c_1 A \]

\[ W_{tot} = c_1 A + \frac{1}{2} kA^2 = \frac{1}{2} m v_{x=0}^2 \]

\[ v_{x=0} = \sqrt{\frac{2}{m} \left( c_1 A + \frac{1}{2} kA^2 \right)} \]
b.) \( V_{x=B} = ? \)

\[ W_{x=0,B}^{F_s} = 0 \]

\[ W_{x=0,B}^{F_1} = \int_0^B c_1 \, dx = c_1 x \bigg|_0^B = c_1 B \]

\[ W_{x=-A,B}^{\text{tot}} = c_1 A + c_1 B + \frac{1}{2} k A^2 = \frac{1}{2} m v_{x=B}^2 - 0 \]

\[ W_{x=0,B}^{\text{tot}} = c_1 B = \frac{1}{2} m v_{x=B}^2 - \frac{1}{2} m v_{x=0}^2 \]

Solve for \( V_{x=B} \). \( \text{Part A} \)
c. \( V(x, y) = D = ? \) \( D: x = L, y = H \)

\[
W_{B,D} = 0
\]

\[
W_{y=0,H}^{F_3} = -mgH
\]

\[
W_{(B,D)}^{F_1} = \int_{0}^{L} c_1 \, dx + \int_{0}^{H} c_2 \, dy
\]

\[
= c_1 x\bigg|_B^L + c_2 y\bigg|_0^H
\]

\[
= c_1 (L - B) + c_2 H
\]

\[
W_{B,D}^{\text{tot}} = -mgH + c_1 (L - B) + c_2 H + c_1 A + c_1 B + \frac{1}{2} k A^2 = \frac{1}{2} m v_D^2 - O
\]

\[\text{Started from rest at } x = -A\]

OR

\[
W_{B,D}^{\text{tot}} = -mgH + c_1 (L - B) + c_2 H
\]

\[
= \frac{1}{2} m v_D^2 - \frac{1}{2} m v_B^2
\]

\[\text{Calculated in part (b)}\]
3) Conservation of Mechanical Energy (only conservative forces)

- May be asked to show that a force is conservative by finding potential energy function
  - \( U(x) = -\int F_x \, dx \)
  - Verify that \( F_x = -\frac{dU}{dx} \)
- Write down \( U(x) \) or \( U(y) \) for each force that is doing work
- Define coordinate system
- Calculate \( E_{tot} = U_i + K_i \) at some point.
- Use \( U_f + K_f = E_{tot} \) to find unknown.
# 4. (old exam)

1-dim force

\[ F_x = \frac{\alpha}{x^3} - \frac{\beta}{x^2} \]

Show that it's conservative by finding potential energy func.

a.) \( U(x) = -\int F_x \, dx \)

\[ = -\int (\alpha x^{-3} - \beta x^{-2}) \, dx \]

\[ = -\alpha x^{-2} - \beta x^{-1} = \frac{\alpha}{2x^2} - \frac{\beta}{x} \]

\[ F_x = -\frac{dU}{dx} = -\left( -\alpha x^{-3} + \beta x^{-2} \right) \]

\[ = \frac{\alpha}{x^3} - \frac{\beta}{x^2} \checkmark \]

\[ \therefore F_x \text{ is conservative} \]
b.) velocity at \( x = \frac{\alpha}{2\beta} \) is \( v_1 \)

Find velocity at \( x = A \)

\[
E_{\text{tot}} = U(x = \frac{\alpha}{2\beta}) + K(x = \frac{\alpha}{2\beta}, v = v_1)
\]

\[
= \frac{\alpha}{2\beta} - \frac{\beta}{\left(\frac{\alpha}{2\beta}\right)} + \frac{1}{2} m v_1^2
\]

\[
E_{x=A} = \frac{\alpha}{2A} - \frac{\beta}{A} + \frac{1}{2} m v_1^2
\]

Solve for \( v_{x=A} \)