Review Sessions

Sunday, March 19
4 pm

Exam
March 21
Ch. 6-10

and
6 pm

in ENPH202

to be confirmed by email

WebCT - extra credit

finish everything (incl. Math Quizzes)

up to \( \frac{3}{2} \) incl. Ch. 90 - Due at 11AM March 20
10 Simple Harmonic Motion

\[ x = A \cos(\omega t) \pm A \]

\[ v = v_0 + at \]

\[ x = v_0 t + \frac{1}{2} at^2 \]

for constant force \( F \)
10.1 The Ideal Spring and SHM

\[ F_A = kx \]

relaxed spring

proportionality constant

\[ [k] = \frac{N}{m} \]

Restoring force \[ F = -kx \]

Hooke's Law
Attach object, compress spring and release $\Rightarrow$ "sinusoidal vibration"

$x = 0$ $\Rightarrow$ $x$

$F > 0$, accelerates

$F = 0$, $v = v_{\text{max}}$

$F < 0$, decelerates

$v = 0$

$X_{\text{max}} = \pm A$ "Amplitude"
10.2 STHM and Reference Circle

- Uniform circular motion w/ angular speed ω
- Project motion on the screen

radius \( r = A \)

\[ \begin{align*}
\theta &= \omega t \\
v_T &= \frac{A\omega}{r} \\
\text{velocity} &\text{ acc. } (a_c = \frac{v_T}{r}) \\
v &= v_T \sin \theta = -v_T \sin(\omega t) = -A\sin(\omega t) \\
a &= -a_c \cos \theta = -\left(\frac{A^2\omega^2}{r^2}\right) \cos(\omega t) \\
a &= -A\omega^2 \cos(\omega t) \\
\text{max} &= A\omega^2
\end{align*} \]
Relation to Hooke's Law

\[ F = ma \Rightarrow -kx = ma \]

\[ -k(A \cos(wt)) = m(-Aw^2 \cos(wt)) \]

\[ -k = -mw^2 \]

\[ \Rightarrow w^2 = \frac{k}{m} \]

\[ T = \frac{1}{f} \]

\[ w = 2\pi f \]

\[ \omega = \text{angular frequency of STHM of mass } m \text{ & spring constant } k \]
10.3 Energy in S+M

elastic potential energy $\Rightarrow$ work done by spring on an object

$$W_{\text{elastic}} = \overrightarrow{F} \cdot \overrightarrow{S} = \overrightarrow{F} (x_f - x_0)$$

$$= \frac{1}{2} k x_0^2 - \frac{1}{2} k x_f^2 = PE_0 - PE_f$$

Spring force is a conservative force

$$W_c = -\Delta PE_{\text{elastic}} = PE_0 - PE_f$$
Total mechanical energy:
\[ E = \frac{1}{2}mv^2 + \frac{1}{2}I \omega^2 + mgh + \frac{1}{2}k\chi^2 \]
\[ E = KE + KE_{\text{rot}} + PE_{\text{grav}} + PE_{\text{elastic}} \]

10.4 Simple Pendulum

\[ \zeta = -mgL\sin\theta \]
\[ \theta \text{ is small} \Rightarrow \sin\theta \approx \theta \]

\[ \zeta = -mgL\theta = I\alpha \]
\[ k'\theta = I\alpha \quad I = mL^2 \]
\[ -k'\theta = mL^2\alpha \]
\[ \omega = \sqrt{\frac{k'}{I}} = \sqrt{\frac{k}{mL^2}} = \sqrt{\frac{g}{L}} = \omega \]
e.g. Pendulum Clock requires \( \frac{1 \text{ rev}}{s} \)

What must be the length \( L \)?

\[
w = \sqrt{\frac{g}{L}} \quad \text{and} \quad w = \frac{2\pi \text{ rad}}{s} \quad \Rightarrow \quad f = \frac{1 \text{ rev}}{s}
\]

\[
2\pi \frac{\text{ rad}}{s} = \sqrt{\frac{g}{L}} \quad \Rightarrow \quad L = 0.248 \text{ cm}
\]

Physical Pendulum - extended swinging object

\[
l_1 = 12 \text{ cm} \quad l_2 = 26 \text{ cm} \quad m_1 = 1.4 \text{ kg} \quad m_2 = 2.5 \text{ kg}
\]

rod \( l_1 = 25 \text{ cm} \), \( m_1 = 1.4 \text{ kg} \)

steel ball \( m_2 = 2.5 \text{ kg} \)

\[
w = \sqrt{\frac{k'}{I}}
\]

\[
I = m_2 l_1^2 + \frac{1}{2} m_1 l_1^2
\]

\[
k' = Mg
\]

\[
M = m_1 + m_2
\]

\[
L = x_{cg} = \frac{m_1 l_1^2 + m_2 l_1}{m_1 + m_2} = 0.205 \text{ m}
\]

\[
w = \sqrt{\frac{k'}{I}} = \sqrt{\frac{7.83 \text{ Nm}^2}{0.185 \text{ kgm}^2}} = 6.51 \frac{\text{ rad}}{s}
\]

\( (2\pi = 6.28) \)
10.5 Damped Harmonic Motion

Ideal:

Damped: Amplitude $A$ of vibration decreases with time.

$$x(t) = A(t) \cdot \cos(\omega t)$$

$$A(t) = A_0 e^{-\gamma t}$$

$\gamma = \text{damping factor}$
10.6 Driven Harmonic Motion

Apply external driving force ⇒ put energy into system

\[ F(t) = F_0 \sin(\omega_D t) \]

\[ \omega_D = \omega_{\text{SHM}} \]

"resonance frequency"
10.7 Elastic Deformation of Materials

Linear

\[ Y = \text{Young's modulus (material constant)} \]

\[ F = Y \frac{\Delta L}{L_0} A \]

\[ \Delta L \propto \frac{L_0}{A} F \]

e.g. Steel \( Y = 2 \times 10^7 \frac{N}{m^2} \)
Volume

e.g. increase air pressure in chamber

\[ \text{pressure} = \frac{F}{A} \]

\[ [P] = \frac{N}{m^2} = 1 \text{Pa} \] (Pascal)

Volume change due to pressure change:

\[ \Delta V \propto -V_0 \Delta P \]

or

\[ \Delta P = -B \frac{\Delta V}{V_0} \]

\( B = \text{bulk modulus} \) (material constant)