Chapter 12: Waves & Standing Waves

Wave:
    disturbance/signal transmitted through a medium (water, string, air) or in vacuum (electromagnetic) carrying energy

Two types:

(i) **transverse**

\[ t_1 > t_0 \]

\[ t_2 > t_1 \]

**disturbance is perpendicular to direction of travel**
(ii) longitudinal

[Diagram of longitudinal waves with arrows indicating direction]

disturbance/compression parallel to direction of travel
Periodic Waves

$t_0 = 0$

amplitude $A$

wavelength $\lambda$

$t = \frac{T}{2}$

$t = T$

$V_{wave}$

after time $T \equiv$ (period) wave has travelled

$\Delta X = \lambda$

$V_{wave} = \frac{\Delta X}{\Delta t} = \frac{\lambda}{T} = \frac{1}{f}$

$f = \frac{1}{T}$
Wave of amplitude $A = 0.013 \text{ m}$

$f = 5.0 \text{ Hz} = 5.0 \frac{\text{cycles}}{\text{s}}$

What is the distance traveled by a particle after $t = 3 \text{s}$.

$d = \frac{4A}{\text{cycles}}$

$d = \frac{4A}{\text{cycle}} \times 5 \frac{\text{cycles}}{\text{s}} \times 3 \text{s}$

$= 60(A) = 0.78 \text{ m}$

$= 78 \text{ cm}$
particle at \(x = 0\) leads to SHM

\[y(t, x=0) = A \sin\left(\frac{2\pi}{T} t\right)\]

\[
\begin{align*}
&\frac{2\pi \cdot T}{4} = \pi/2 \\
&\frac{2\pi \cdot T}{4} = \pi/4
\end{align*}
\]

\[\begin{align*}
-A &< A \sin(x) < A \\
-1 &< \sin x < 1
\end{align*}\]
general:

\[ y_+(t,x) = A \sin \left( \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} \right) \]

\[ y_-(t,x) = A \sin \left[ 2\pi f (t - \frac{x}{u}) \right] \]

wave is traveling in +x direction \( \lambda = \frac{u}{f} \)

\[ u = \lambda f \]

\[ y_+(t,x) = A \sin \left[ 2\pi f (t + \frac{x}{u}) \right] \]

traveling in -x direction
guitar string of mass $m$, length $L$ under tension $F_t$

$v_{\text{wave along string}} = \sqrt{\frac{F_t}{\left(\frac{m}{L}\right)}}$

$\rho = \frac{m}{L}$ mass density
\[ \begin{align*}
\text{equilibrium} \\
\mu &= \frac{m}{L} = 0.0250 \text{ kg/m} \\
V_{\text{wave}} &= 75.0 \text{ m/s} \\
F_r &= m_2 g \\
F_r &= \frac{m_1 g \sin 30^\circ}{0.025 \text{ kg/m}} \\
V_{\text{wave}} &= \sqrt{\frac{F_r}{m/L}} \\
F_r &= 140.6 \text{ N} \\
F_r &= m_2 g \\
m_2 &= 14.3 \text{ kg} \\
m_1 &= \frac{F_r}{g \sin 30^\circ} = 28.7 \text{ kg}
\end{align*} \]
Standing Waves

(i) Signal Reflection

(ii) Period Wave

possible stable wave lengths with 2 fixed ends

"Standing wave"

$L = \frac{\lambda}{2}, \lambda \neq \frac{3\lambda}{2}, \ldots, \ldots$

"harmonic" frequencies:

$L = \frac{n}{2} \frac{\lambda_n}{\lambda_n}$

$\lambda_n = \frac{2L}{n} \Rightarrow f_n = \frac{V}{\lambda_n} = n\left(\frac{V}{2L}\right)$

$n = 1 \Rightarrow \text{"first harmonic"}, \text{"natural or fundamental freq"}$

$n = 2 \Rightarrow \text{second harmonic} \ldots$