Final Exam Topics

**Generalized coordinates and the Lagrangian**
- Use the Euler-Lagrange equation to determine the equations of motion for a given Lagrangian.
- Use the “Principle of Least Action” to derive Lagrange’s equations.
- Show that the Lagrangian is defined only to within an additive total time derivative of any function of coordinates and time.

**Conserved quantities (constants of motion)**
- Use symmetries to find conserved quantities
- Cyclic variables
- Use conserved quantities to reduce the equation of motion to quadratures
- Poisson Brackets in the context of conserved quantities (using the Hamiltonian)

**Hamilton’s Equations and Canonical Transformations**
- Derive Hamilton’s equations
- Calculate the Hamiltonian from a Lagrangian
- Derive any of the 4 types of generating functions
- Determine whether a transformation is canonical
- Find a transformation from a generating function
- Solve for the motion using a canonical transformation
- Conservation of phase-space volume in a canonical transformation

**Hamilton-Jacobi Method**
- What is the nature of the canonical transformation generated by Hamilton’s Principle Function?
- Determine what form of a potential allows separation of variables for a particular choice of coordinates
- Write down the Hamilton-Jacobi equation and reduce it to quadratures using separation of variables.

**Central Force Problem**
- Sketch the “effective” potential as a function of $r$.
- Describe the types of motion possible for different values of the energy.
- Calculate the orbit $r(\phi)$ for a particle in a given (central) potential.
- Calculate the frequency/period of an orbit.
- Calculate the frequency of small oscillations about a circular orbit, $\omega_r$.
- Kepler’s Laws
Scattering
- Disintegration (Decay)
- Elastic Collisions
- CM ↔ Lab Reference Frame Conversions
- Scattering Angle Calculated from Interaction Potential
- Calculation of Cross Section
- Specific Case of Coulomb interaction → Rutherford formula

Small Oscillations
- Expansion of potential about stable equilibrium → equation of motion for small oscillations
- Write Lagrangian for small oscillations about equilibrium (small-angle approximation)
- Forced oscillations and resonance behavior in such a system
- Coupled systems (small oscillations with more than 1 degree of freedom)
- Damped, forced oscillations and resonance behavior
- Parametric Resonance
- Anharmonic Oscillations (keeping more terms in expansion of potential) and how to solve using perturbation theory

Rigid Body Motion
- Calculating the Inertia Tensor - Calculating principal moments of inertia (diagonalizing the inertia tensor)
- Deriving Equations of motion by converting Lagrangian in body-fixed coordinate system to space-fixed coordinate system (Euler angles)
- Deriving Equations of motions from $\frac{d\vec{M}}{dt}_{\text{space-fixed}} = \vec{K}$ and the conversion of the rate of change of a vector from space-fixed to body-fixed coordinate system.
- Using conserved quantities to map out the motion of the terminus of $\vec{M}$ of a free asymmetrical top.
- Rigid Bodies in contact, constraints of coordinates and/or velocities
- Derive Lagrange’s Equation (with Lagrange Multipliers) for a system with constraints
- Non-inertial reference frames: equation of motion and fictitious forces