Constant parabolic coordinate contours

\[ \xi = r + z \]
\[ \eta = r - z \]  \( \phi \) the normal azimuthal angle

Note: \( \xi = 0 \) is the -z axis.
\( \eta = 0 \) is the +z axis.
Parabolic coordinate relationships:

Distance scale factors:

\[ h_\xi = \sqrt{\frac{\xi + \eta}{4\xi}} \quad h_\eta = \sqrt{\frac{\xi + \eta}{4\eta}} \quad h_\phi = \sqrt{\xi \eta} \]

imply the Laplacian takes the form:

\[ \nabla^2 = \frac{4}{\xi + \eta} \left[ \frac{\partial}{\partial \xi} \left( \frac{\xi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( \eta \frac{\partial}{\partial \eta} \right) \right] + \frac{1}{\xi \eta} \frac{\partial^2}{\partial \phi^2} \]